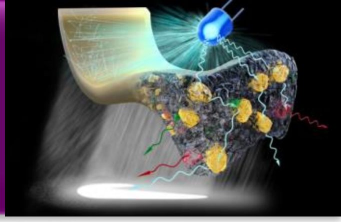
Fundamentals of free-form optics for industrial applications (Workpackage B)

Aurèle ADAM

TUDelft

Goal



Automatic design of a free form for a specific source to a specific illumination

ASML

Offer study case

Signify

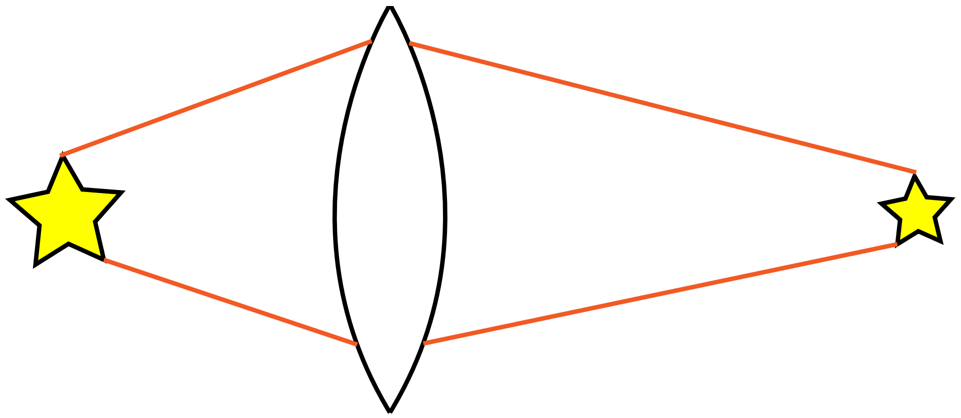
Offer study case & Expertise in optical design

TNO

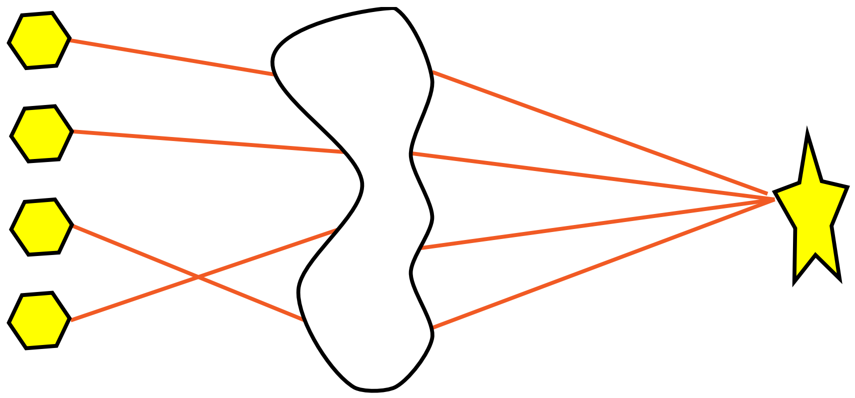
Support in design tool and fabrication

Why is it complicated?

Imaging problem



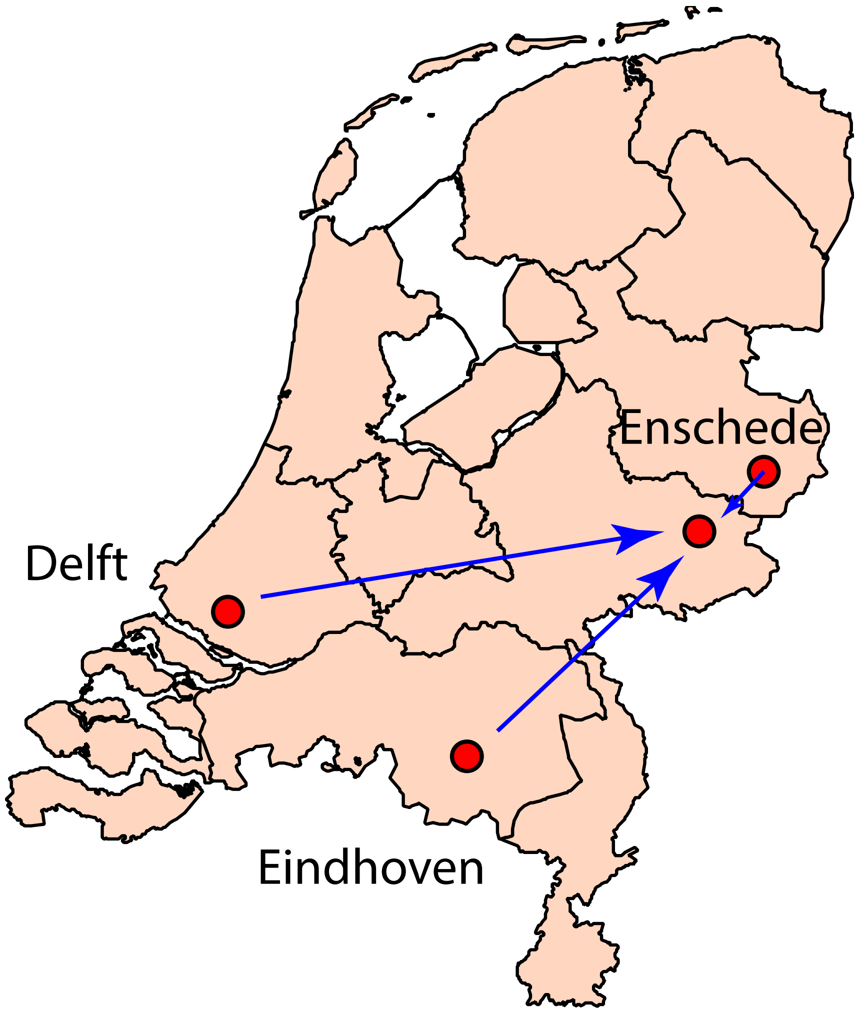
Illumination problem



Differences:

- Multi/extended sources
- No coherence between the source
- Each source contribute to the illumination

Transportation theory



Gaspard Monge

André Marie Ampère

2 medal fields: Villani (2010) and Figalli (2018)

Method of attack

- Analytical by solving the Monge Ampère-Equation:
  - B1. Monge-Ampère Elliptic
    - TU Eindhoven (Lotte Romijn)
  - B2. Monge-Ampère Hyperbolic
    - TU Eindhoven (Maikel Bertens)
- Solving using phase (wavefront)
  - B3. Complex optical surfaces
    - TU Delft (Alex Heemels)

EINDHOVEN UNIVERSITY OF TECHNOLOGY

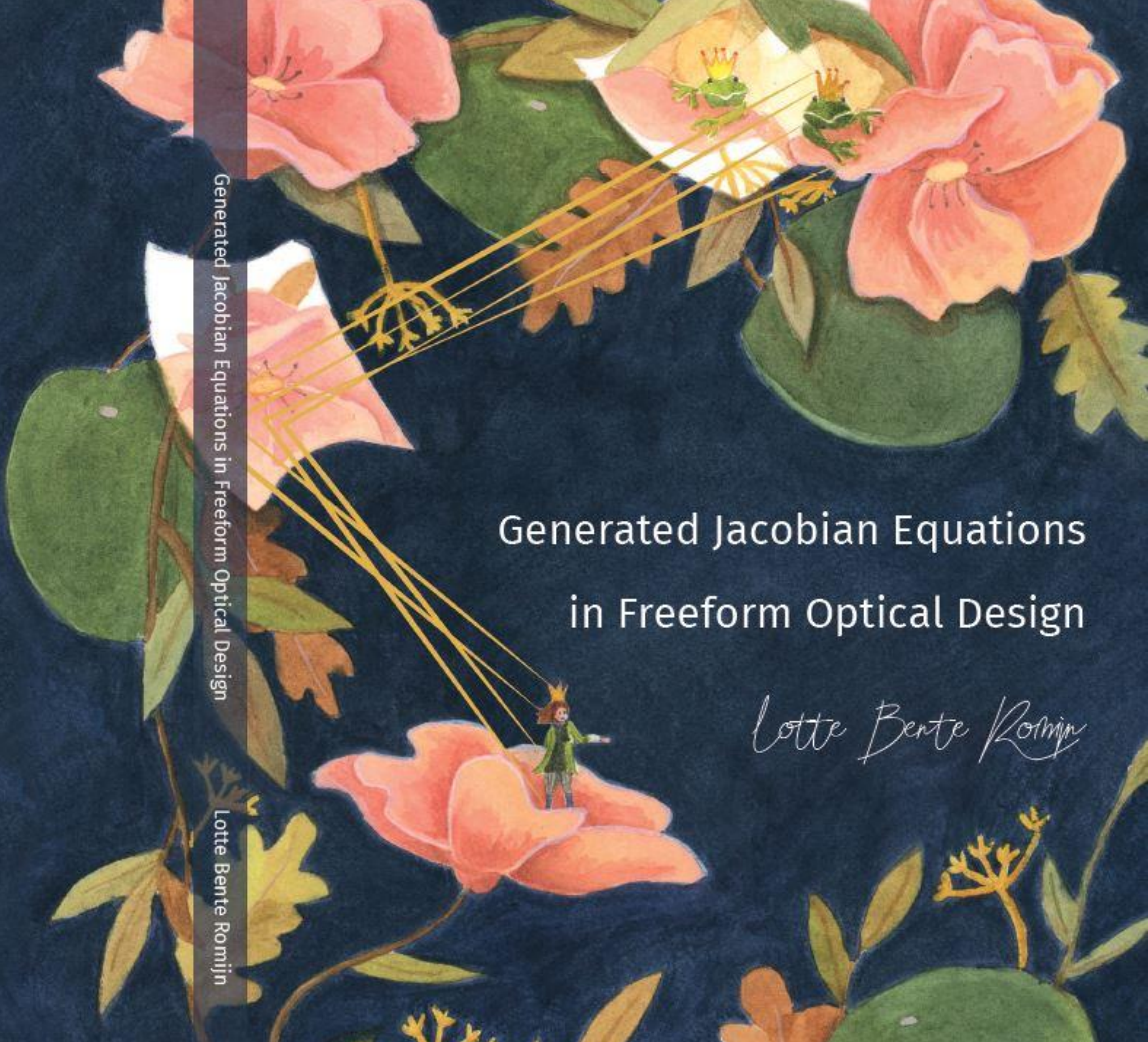
Delft University of Technology

Lotte Romijn - Workpackage B1

Generated Jacobian Equations in Freeform Optical Design (Workpackage B1)

Lotte Romijn

TU Eindhoven

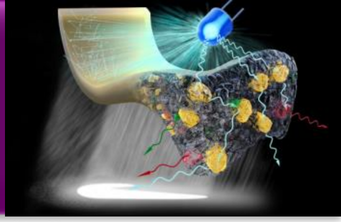


Generated Jacobian Equations in Freeform Optical Design

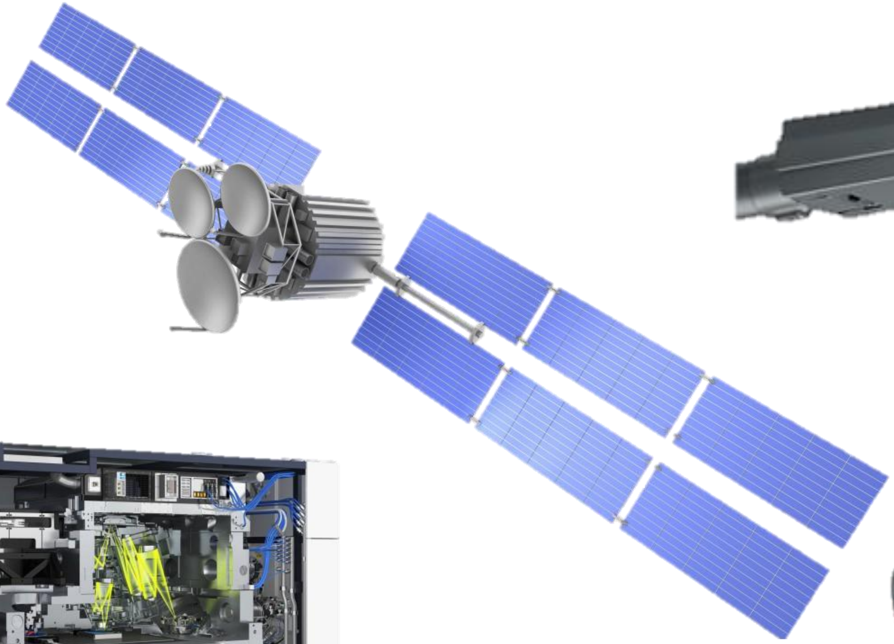
Lotte Bente Romijn

Generated Jacobian Equations in Freeform Optical Design
Lotte Bente Romijn

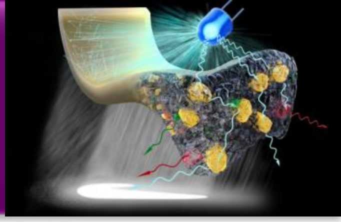
Let there be light



How to transport light efficiently from A to B?

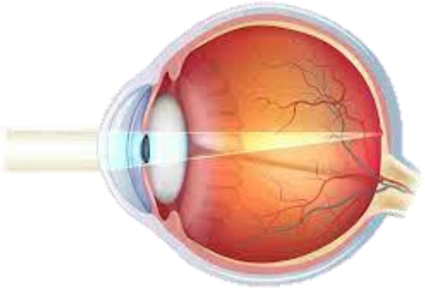


Illumination Optics



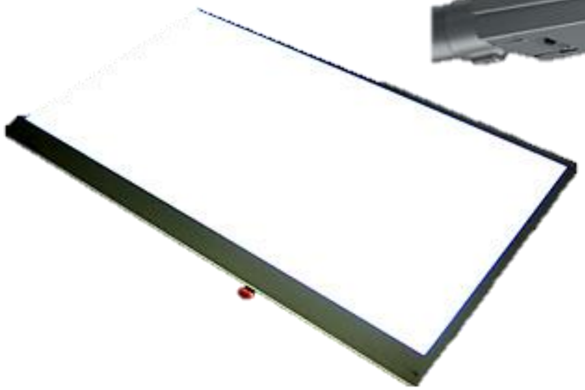
Imaging optics

Form a perfect image of object

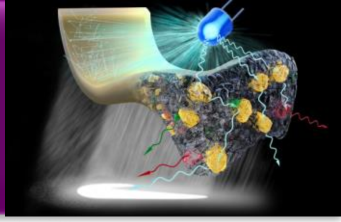


Nonimaging optics

Energy transport from A to B



Freeform non-imaging Optics

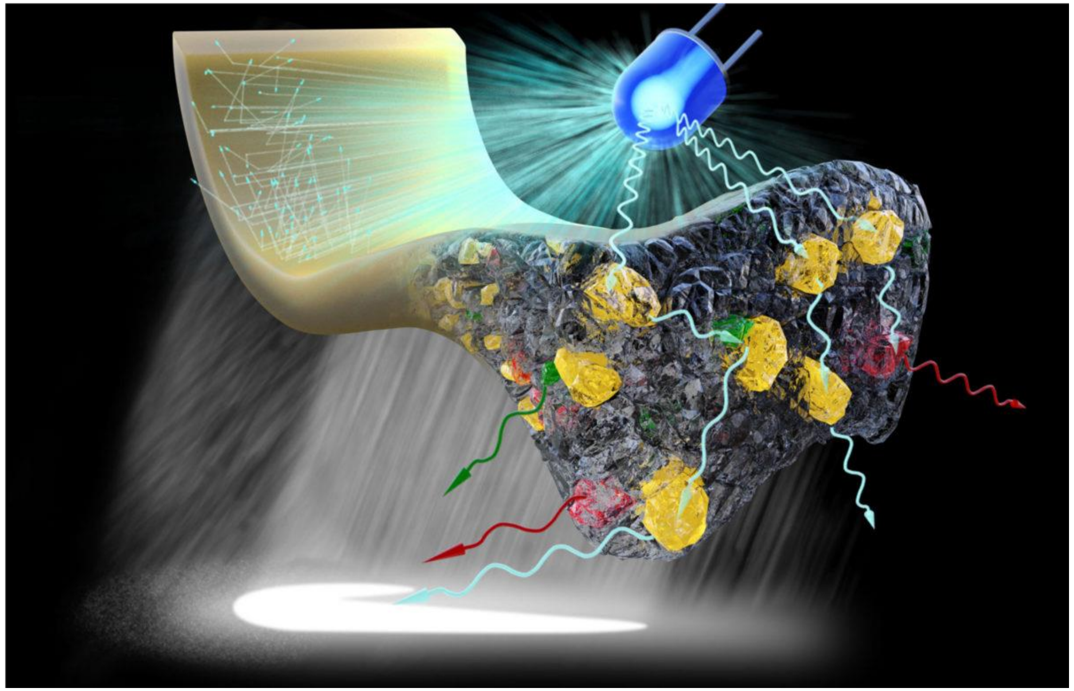


LED light sources:

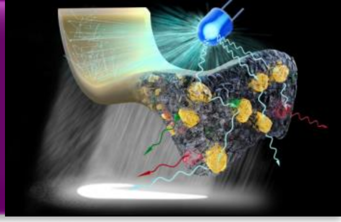
- Energy efficient (20 times)
- Lower temperatures (25°C)

- Plastic materials: reflector/lens

Goal: Compute *freeform* reflectors & lenses



Reflectors and lenses redirect light rays

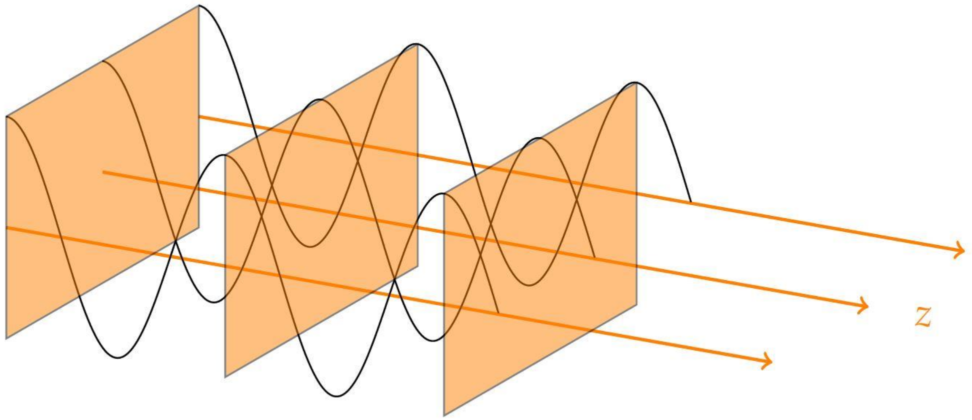


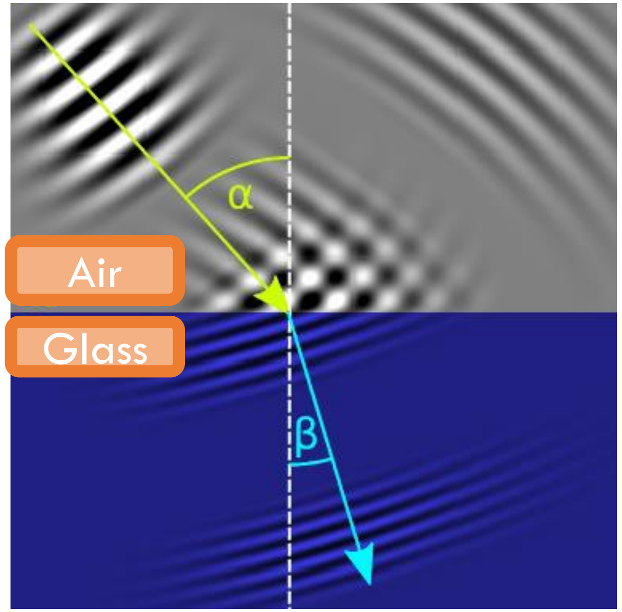
Electromagnetic waves → *Geometrical optics limit*

Maxwell's equations

Ray optics

Law of reflection/refraction

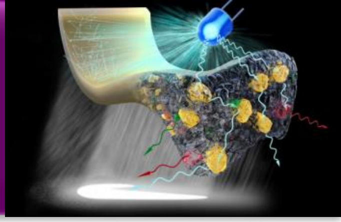




Reflected ray

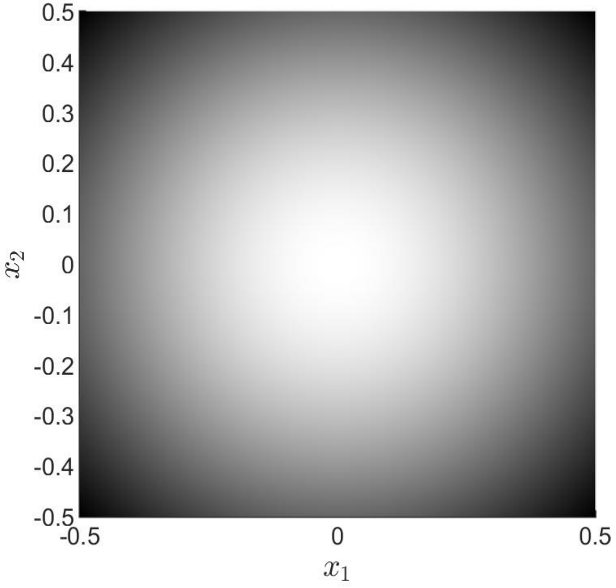
Refracted ray

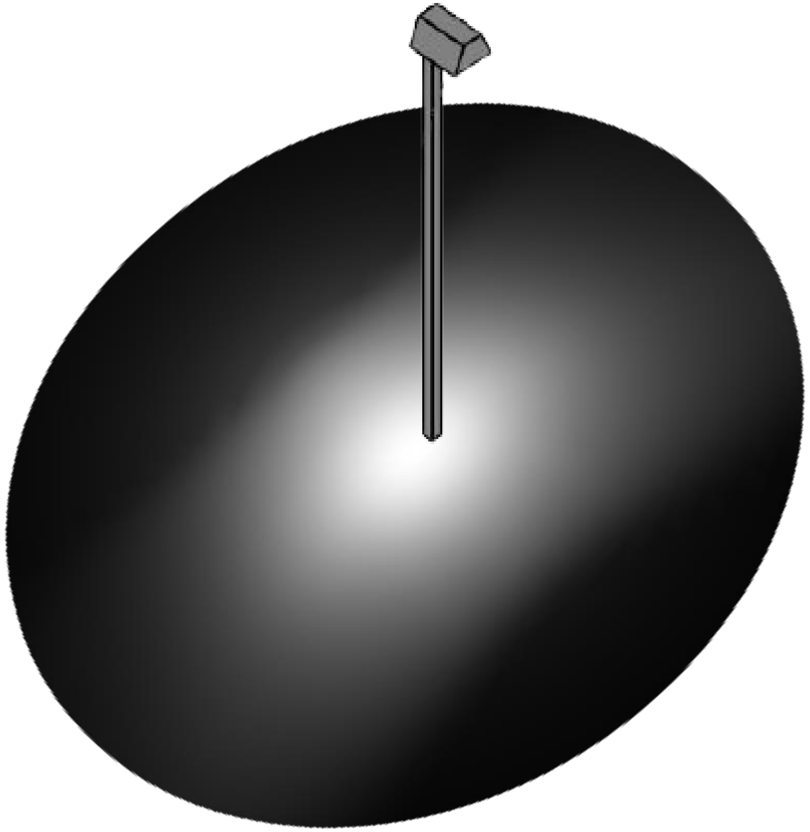
How to compute reflector and lens surfaces?



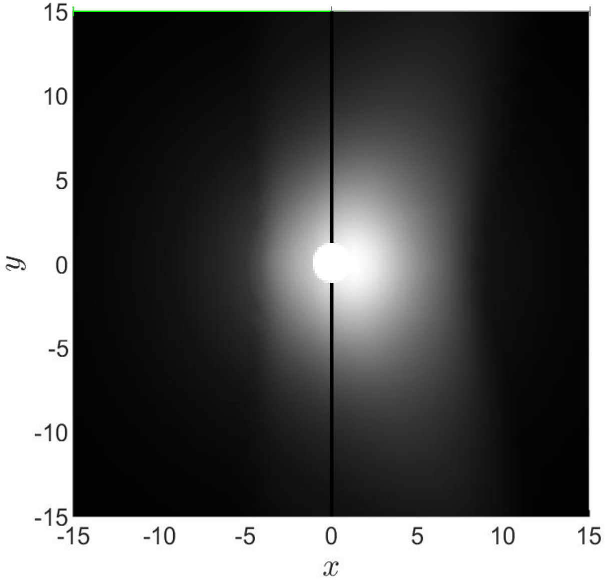
Freeform lens for streetlighting?

Given light source

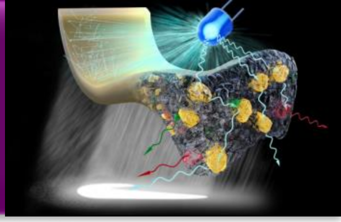




Desired target



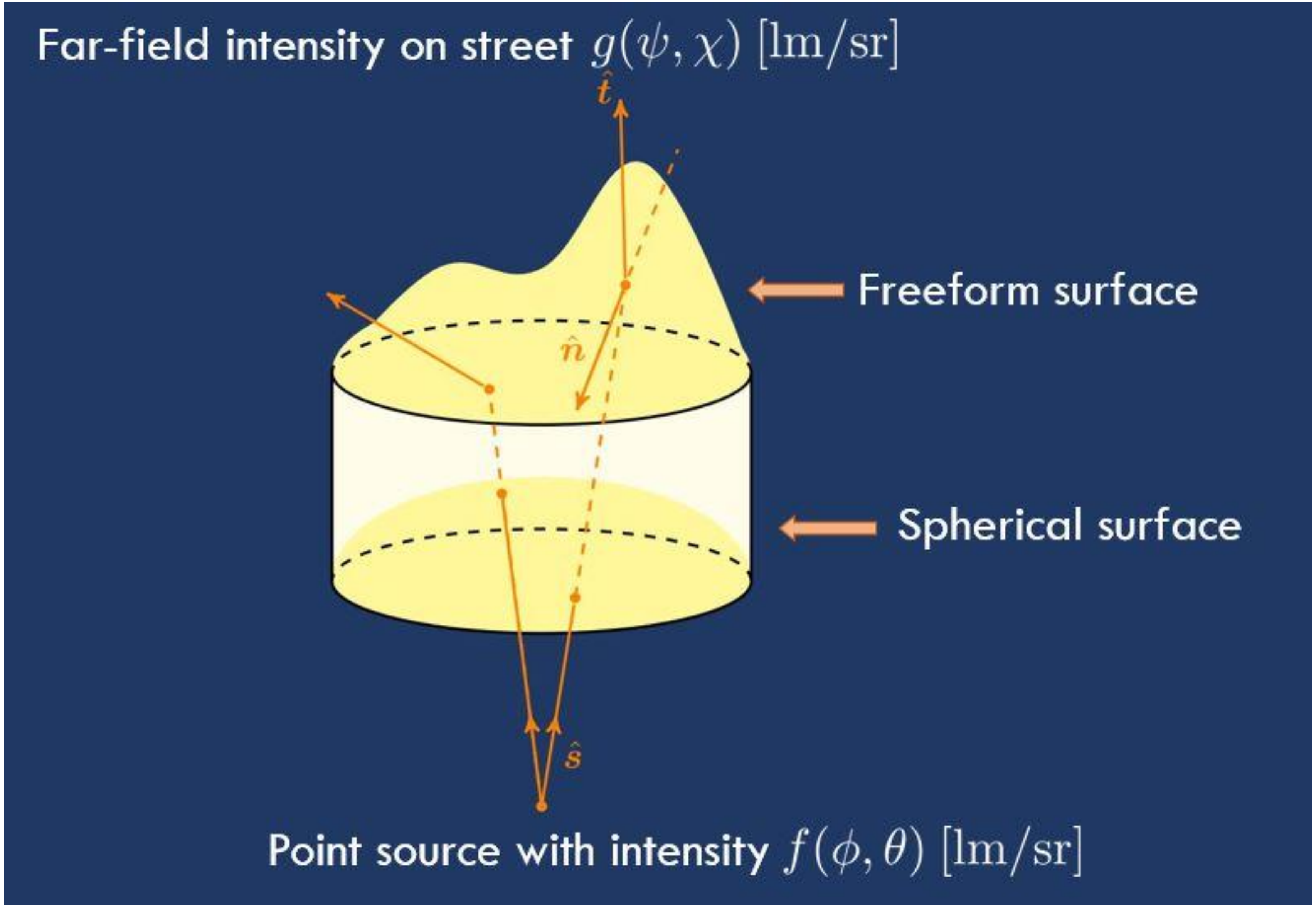
Contributions of my research



1. Generic framework to derive generated Jacobian equations for the 16 base-case optical systems

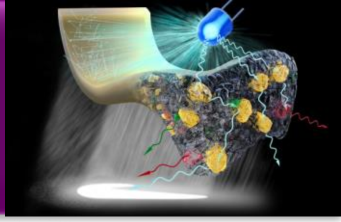
2. Least-squares numerical algorithm

3. Extension to more complicated optical systems



Far-field intensity on street $g(\psi, \chi)$ [lm/sr]
$\hat{t}$
Freeform surface
$\hat{n}$
Spherical surface
$\hat{s}$
Point source with intensity $f(\phi, \theta)$ [lm/sr]

The mathematics



- Laws of geometrical optics

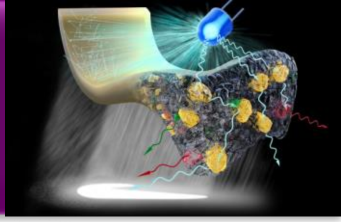
  Gives rise to an **optical mapping** $\boldsymbol{y} = \boldsymbol{m}(\boldsymbol{x})$ which is a function of the source coordinates $\boldsymbol{x}$, surface $u(\boldsymbol{x})$ and $\nabla u(\boldsymbol{x})$

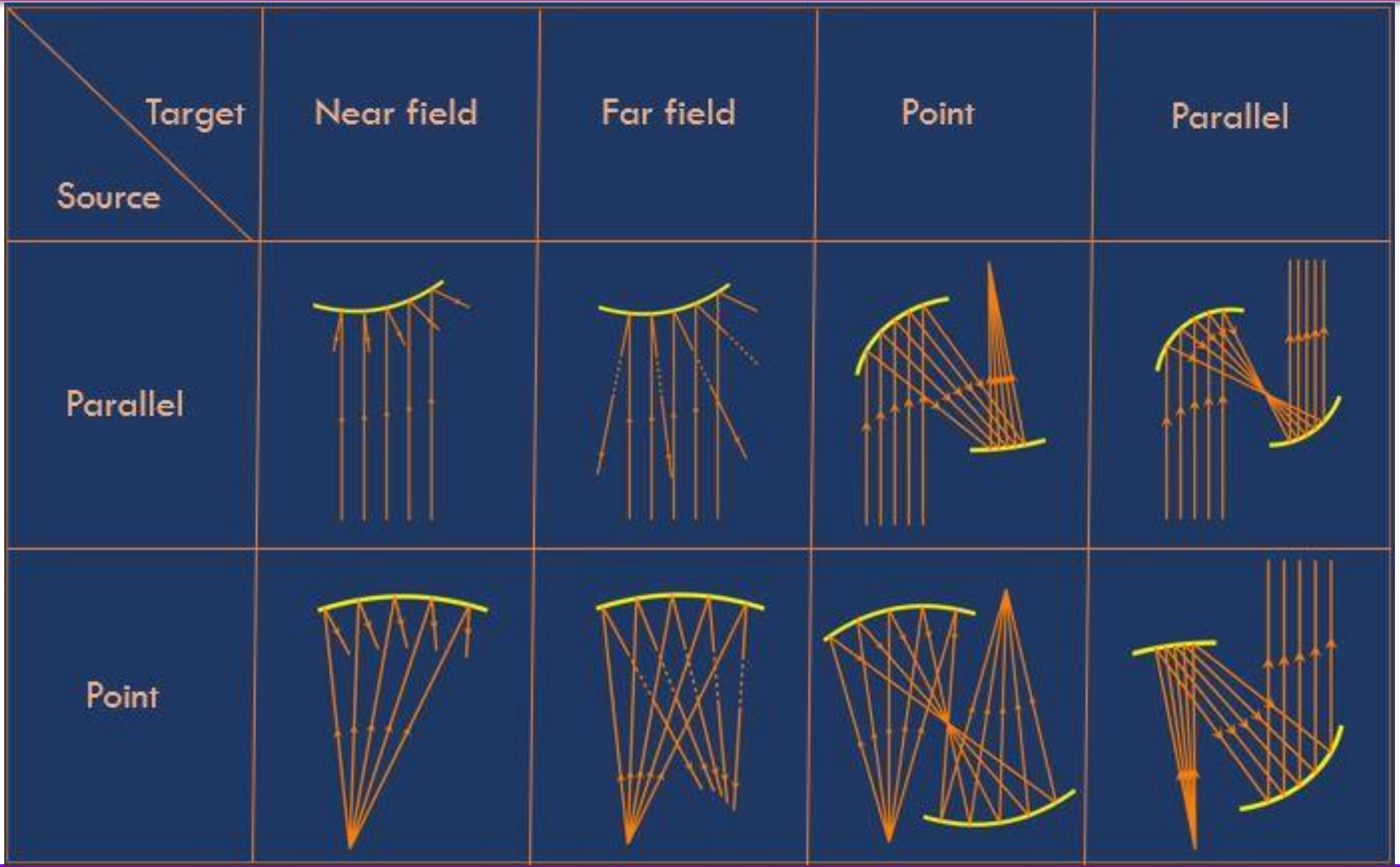
- Energy conservation

$$\det(\mathrm{D}\boldsymbol{m}) = \frac{f(\boldsymbol{x})}{g(\boldsymbol{m}(\boldsymbol{x}))}$$

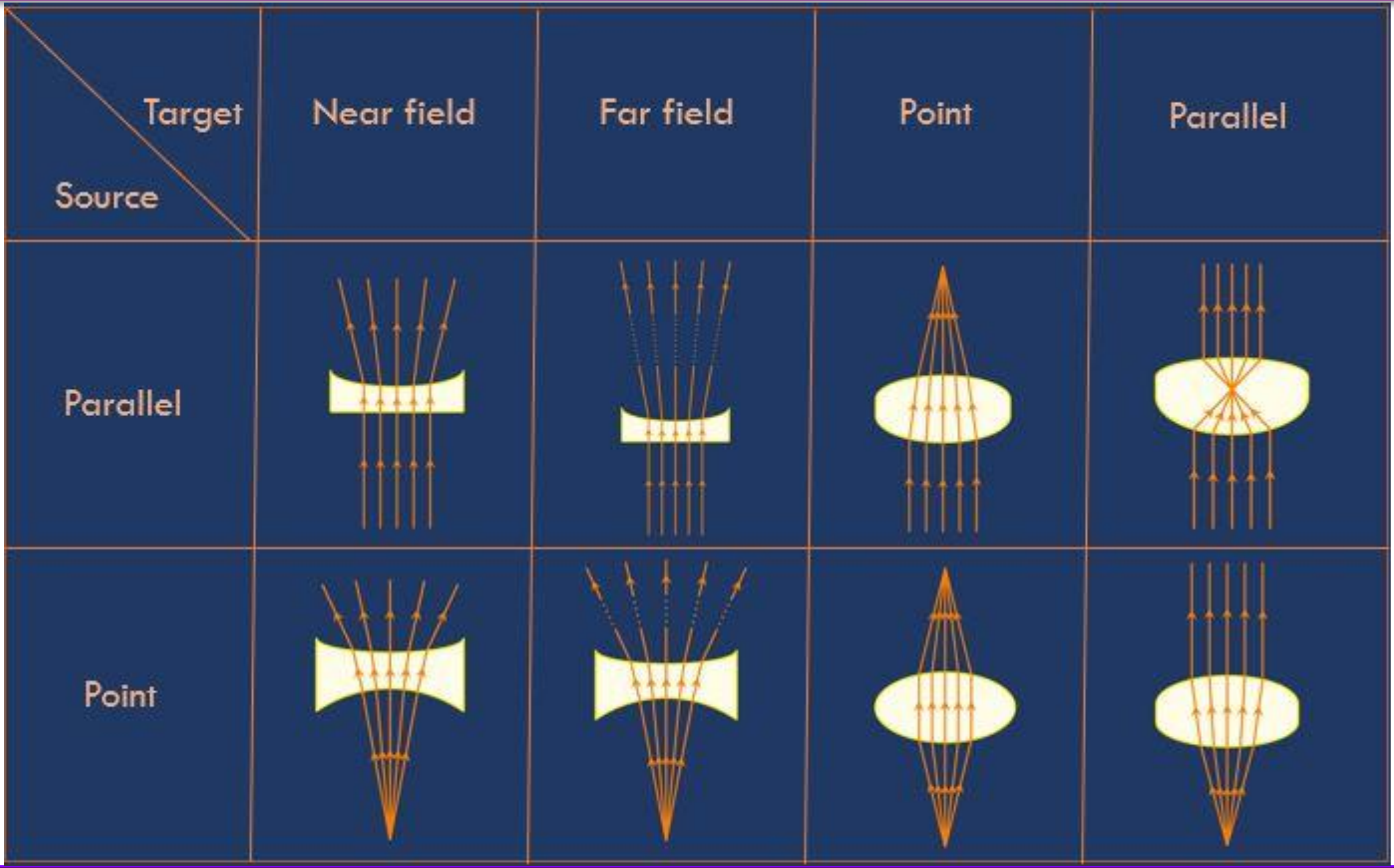
$$\boldsymbol{m}(\partial\mathcal{X}) = \partial\mathcal{Y}$$

16 base cases: Reflector systems

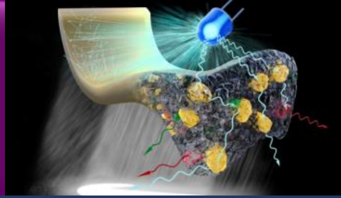




16 base cases: Lens systems

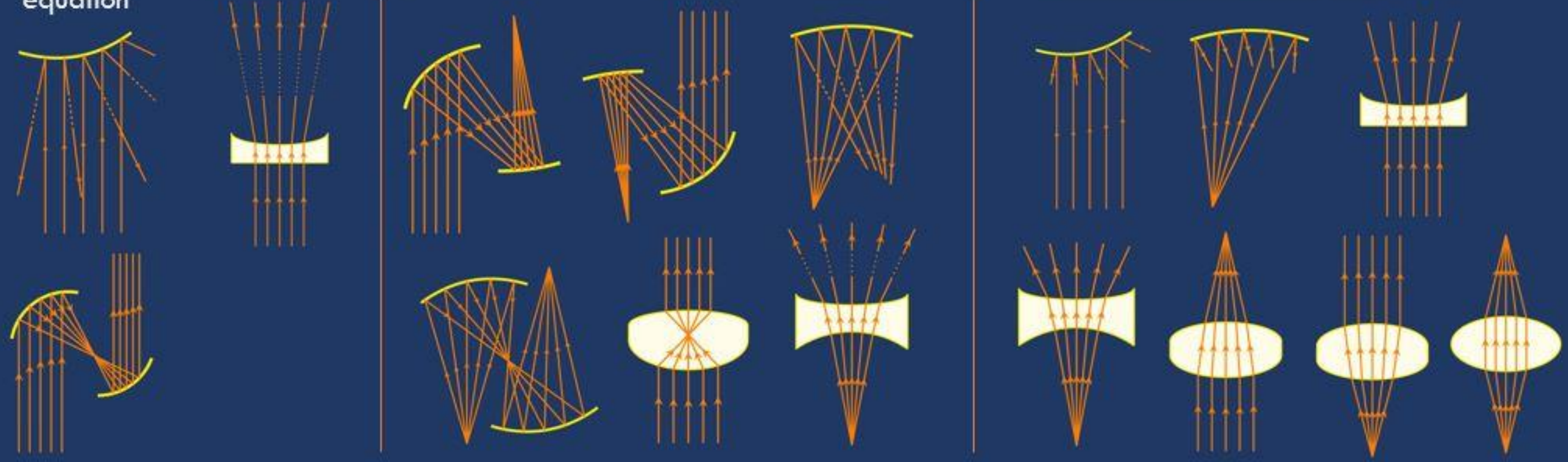


$$\det(\mathrm{D}\boldsymbol{m}) = \frac{f(\boldsymbol{x})}{g(\boldsymbol{m}(\boldsymbol{x}))}$$



$$\det(\mathrm{D}^2 u) = \frac{f(\boldsymbol{x})}{g(\boldsymbol{m}(\boldsymbol{x}))} \qquad \det(\mathrm{D}^2 u_1 + \mathrm{D}_{\boldsymbol{x}\boldsymbol{x}} c) = \frac{f(\boldsymbol{x})}{g(\boldsymbol{m}(\boldsymbol{x}))} \qquad \det(\mathrm{D}^2 u - \mathrm{D}_{\boldsymbol{x}\boldsymbol{x}} G) = \frac{f(\boldsymbol{x})}{g(\boldsymbol{m}(\boldsymbol{x}))}$$

Standard Monge-Ampère equation $\subset$ Generalized Monge-Ampère equation $\subset$ Generated Jacobian equation

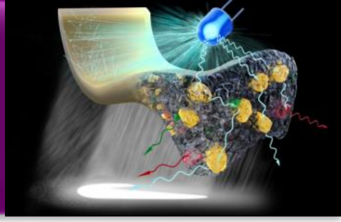


Lotte Romijn, Eindhoven University of Technology, Generated Jacobian equations in freeform optical design

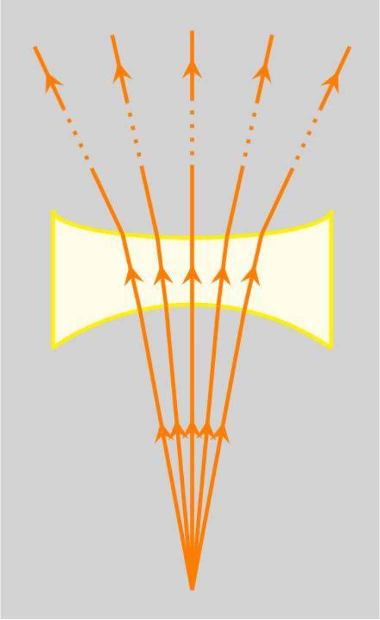
Least-squares algorithm

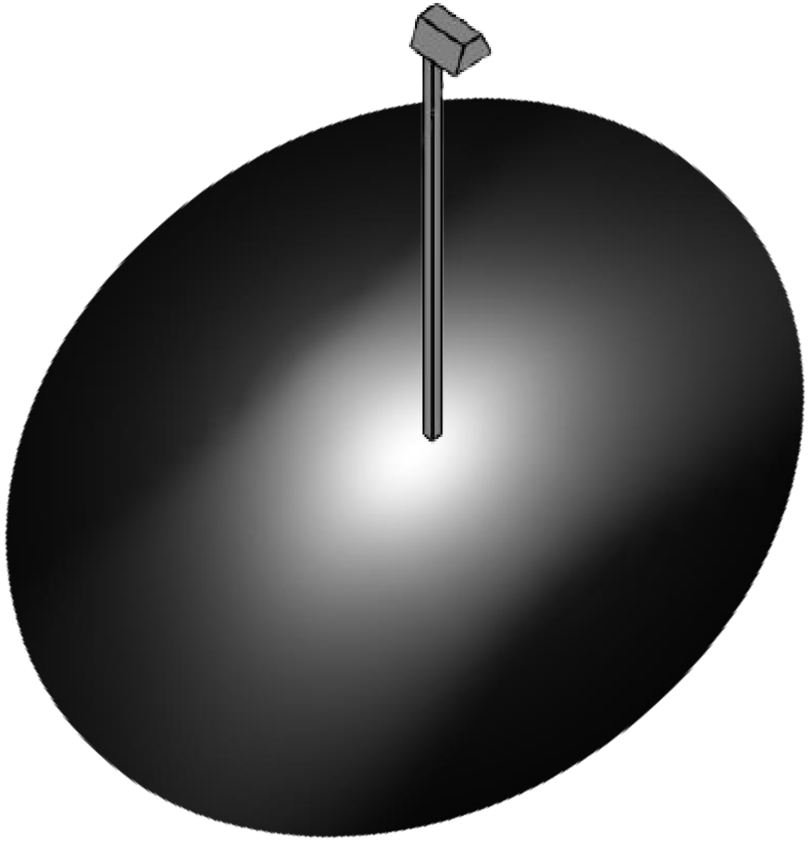
- Start with an initial mapping
  1. Impose energy conservation by minimizing an integral to solve the nonlinear PDE in the **domain**
  2. Impose the boundary condition by minimizing an integral for the deviation from the current mapping to the target **boundary**
  3. Compute the new **mapping** by solving a coupled boundary value problem
  4. Compute the new **surface** by solving a Neumann problem
  5. Repeat

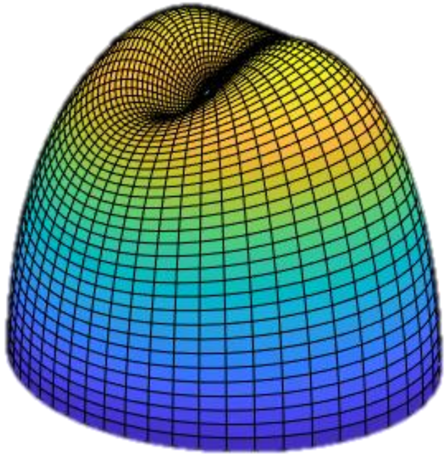
Peanut Lens



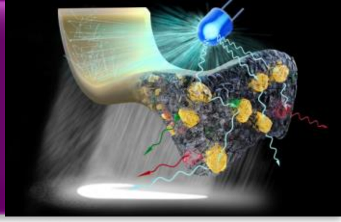
Point to far field



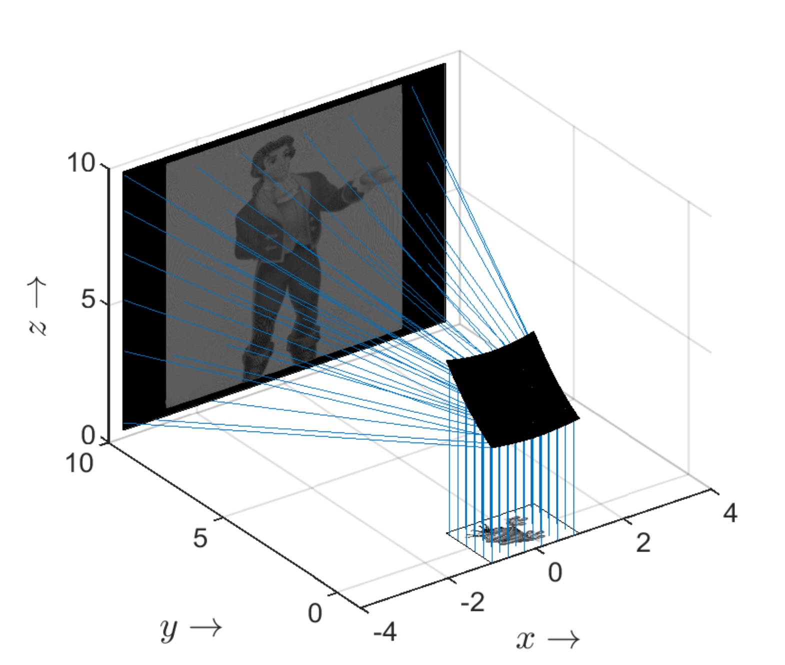




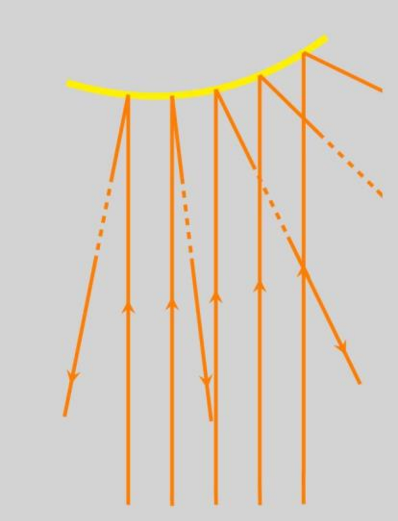
Frog to Prince



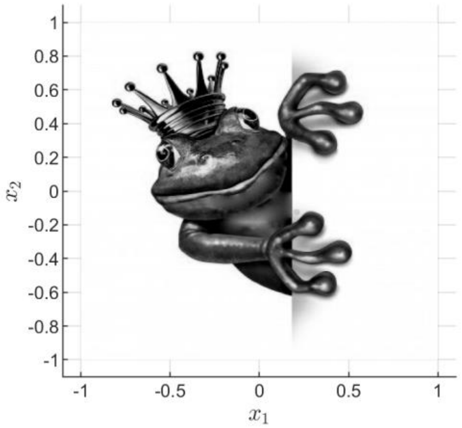
Reflector converting a parallel beam into a prince on a screen in the far field



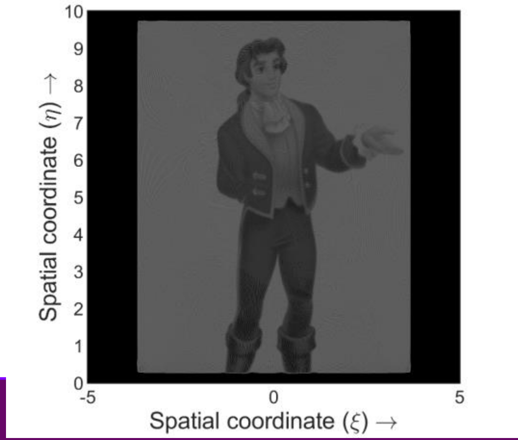
Parallel to far field



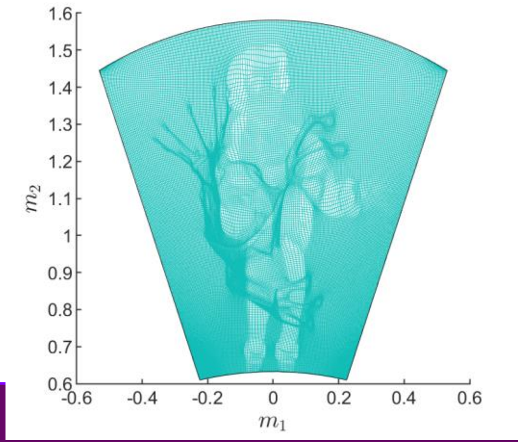
Source emittance



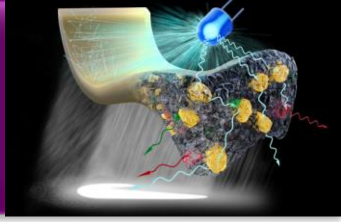
Ray tracing result



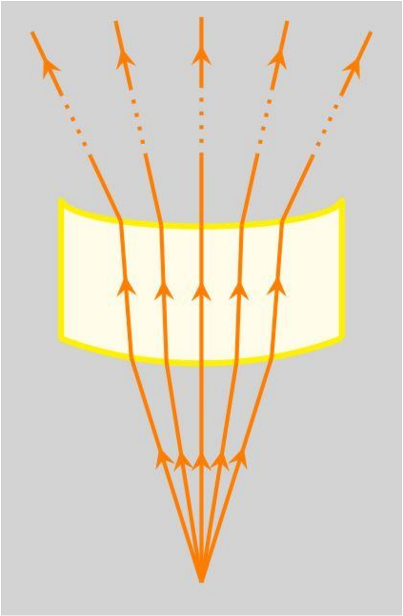
Mapping from source to stereographic target



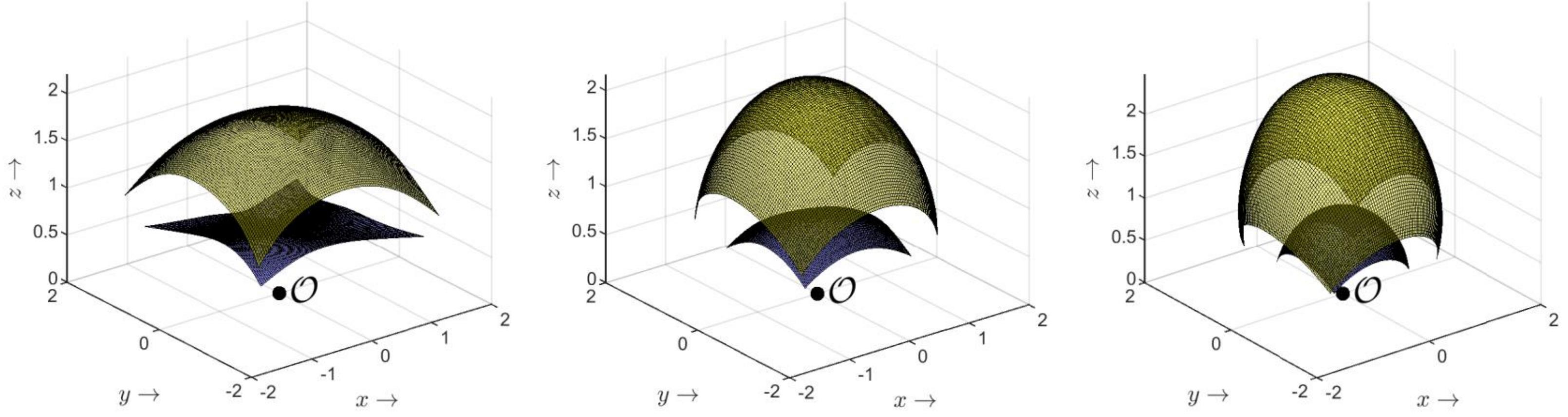
Double Freeform Lens



Point to far field double freeform lens



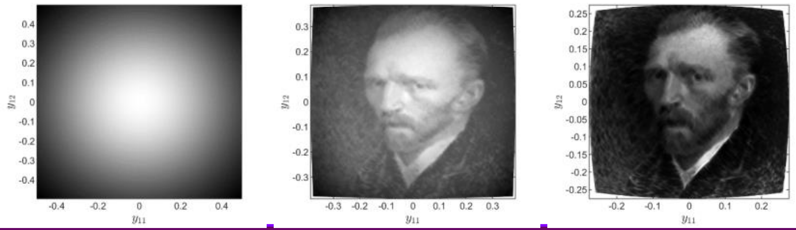
Multiple designs



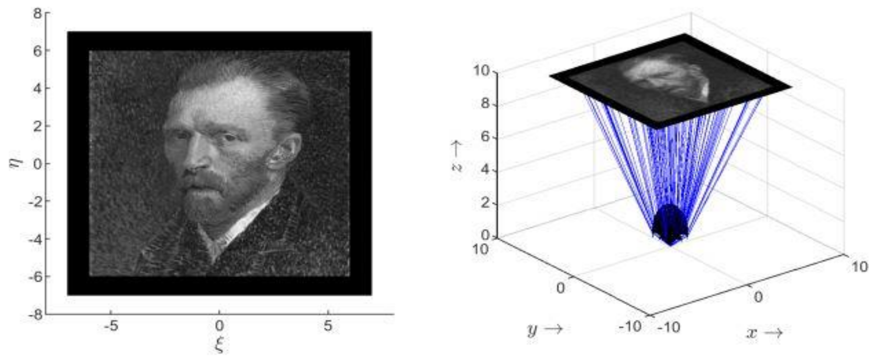
Choose an intermediate target intensity

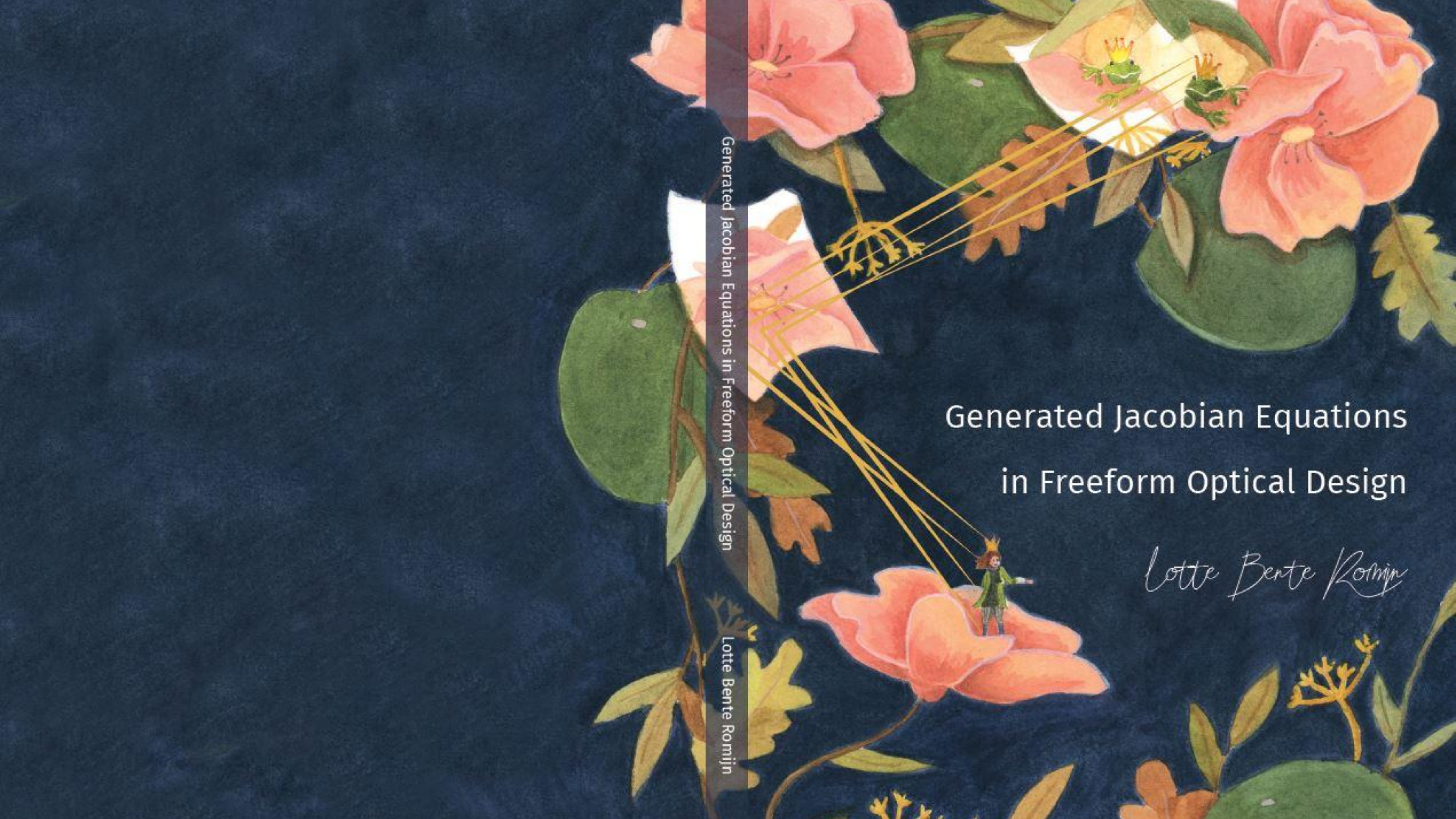
Source

Final stereographic target



Ray tracing results





Generated Jacobian Equations in Freeform Optical Design

Lotte Bente Romijn

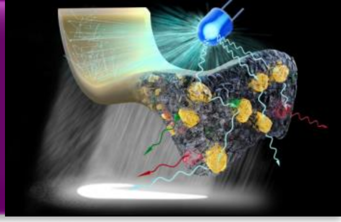
Maikel Bertens – Workpackage B2

A Least Squares Solver for the Hyperbolic Monge-Ampère Equation

Maikel W. M. C. Bertens

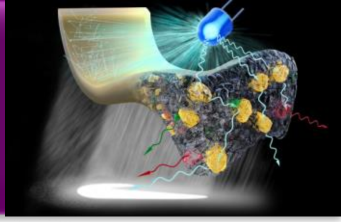
Eindhoven University of Technology, Eindhoven, The Netherlands

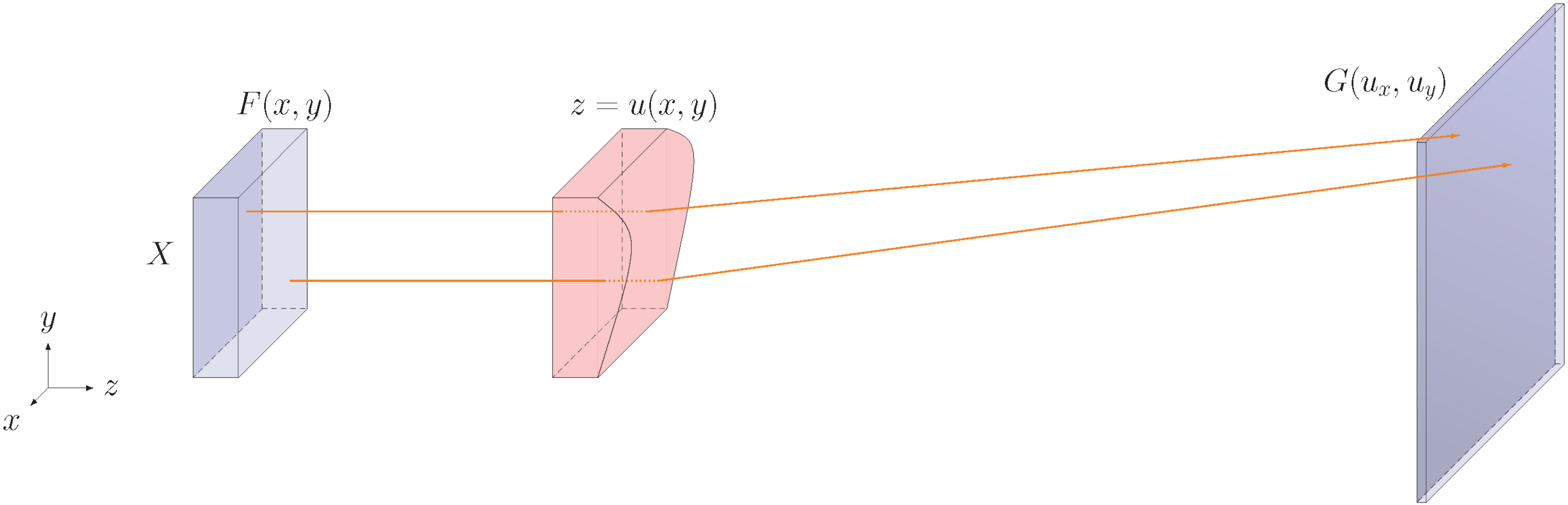
Contents



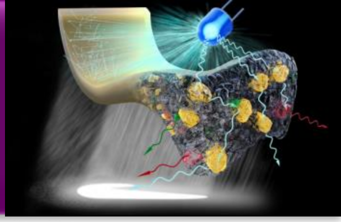
- Optical and mathematical model
- Elliptic versus hyperbolic solutions
- Least-squares solver
- Numerical results
- Conclusions

Optical Model





Optical Model



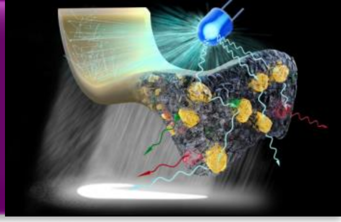
$$u_{xx}u_{yy} - u_{xy}^2 = \pm\frac{F(x,y)\left(u_x^2 + u_y^2 + 1\right)}{4\,G\left(u_x, u_y\right)}, (x,y) \in X$$

$$F(x,y) > 0, (x,y) \in X, \qquad G\left(u_x, u_y\right) > 0, \left(u_x, u_y\right) \in Y$$

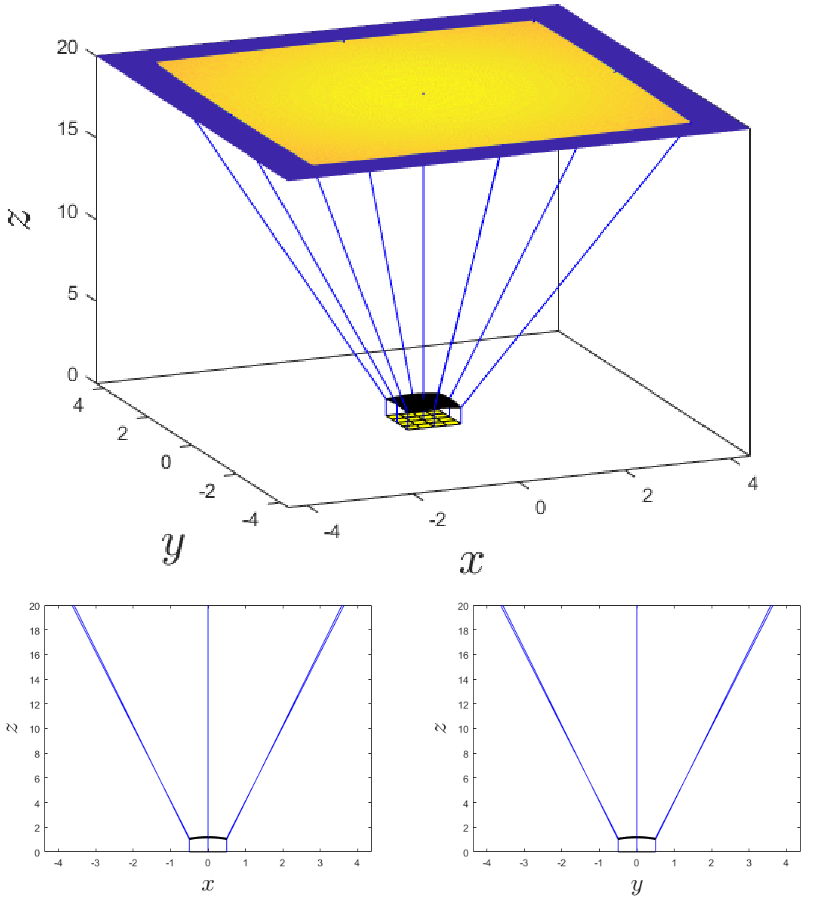
$$\boldsymbol{m} = \nabla \mathrm{u}, \qquad \mathbf{m}(\partial X) = \partial Y$$

\+ Elliptic – Hyperbolic

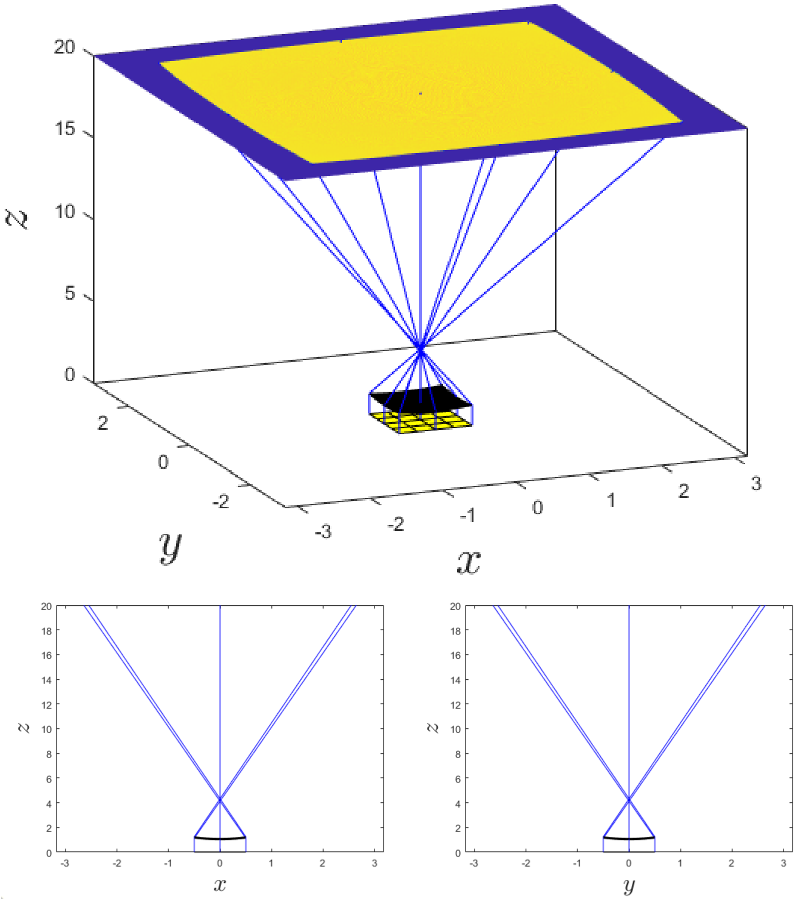
Raytrace Comparison



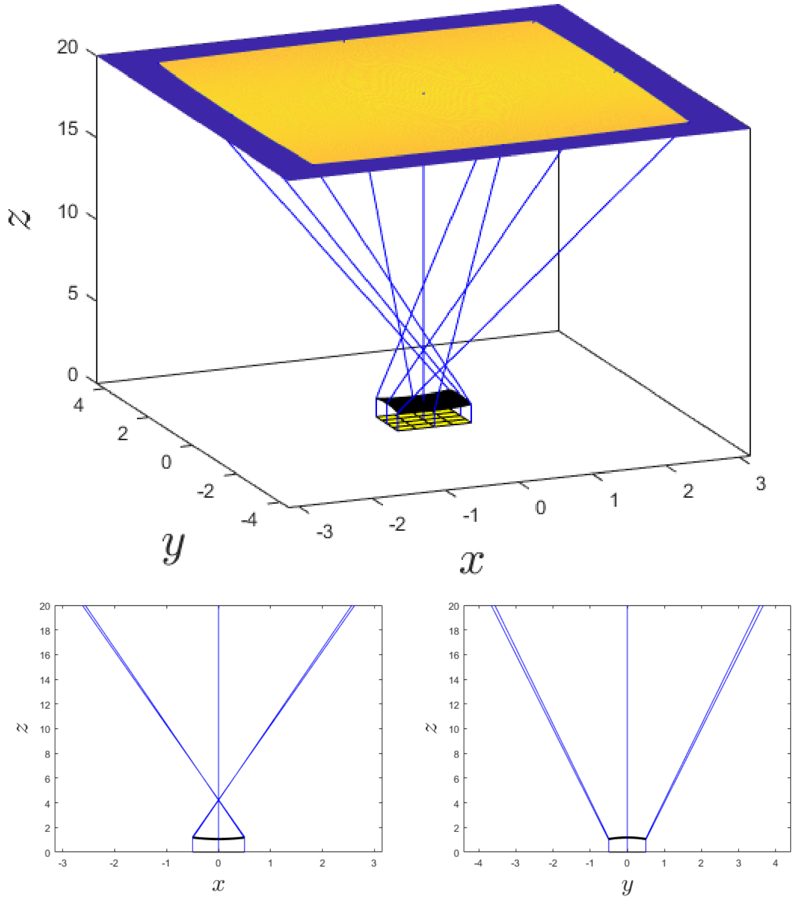
Elliptic (convex)



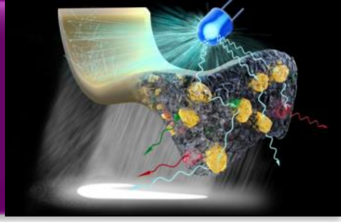
Elliptic (concave)



Hyperbolic



Least-Squares Solver

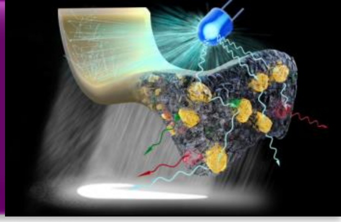


$$|\boldsymbol{Dm}| = u_{xx}u_{yy} - u_{xy}^2 = \pm f^2, \qquad \boldsymbol{m} = \nabla u, \qquad \boldsymbol{m}(\partial X) = \partial Y$$

Iteratively minimize:

- Interior error: $J_I(\boldsymbol{m}, \boldsymbol{P}) = \frac{1}{2}\iint_X \|\nabla(\boldsymbol{m}^{\mathrm{T}}) - \boldsymbol{P}\|^2 \mathrm{d}\boldsymbol{x}$
- Boundary error: $J_b(\boldsymbol{m}, \boldsymbol{b}) = \frac{1}{2}\oint_{\partial X} |\boldsymbol{m} - \boldsymbol{b}|^2 \, \mathrm{d}s$
- Total weighted error: $J(\boldsymbol{m}, \boldsymbol{P}, \boldsymbol{b}) = \alpha J_I(\boldsymbol{m}, \boldsymbol{P}) + (1-\alpha) J_b(\boldsymbol{m}, \boldsymbol{b})$

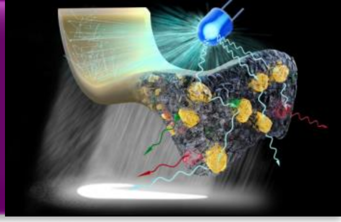
Least-Squares Solver



Minimization of:

- $J_I(\boldsymbol{m}, \boldsymbol{P})$: Solve a nonlinear system of equations in 4 variables
    - $\det \boldsymbol{P} > 0$, $\boldsymbol{P}$ positive/negative definite
    - $\det \boldsymbol{P} < 0$, $\boldsymbol{P}$ indefinite
- $J_b(\boldsymbol{m}, \boldsymbol{b})$: orthogonal projection of $\boldsymbol{m}$ onto ∂Y
- $J(\boldsymbol{m}, \boldsymbol{P}, \boldsymbol{b})$: solve two decoupled Poisson equations with Robin boundary conditions

Numerical results

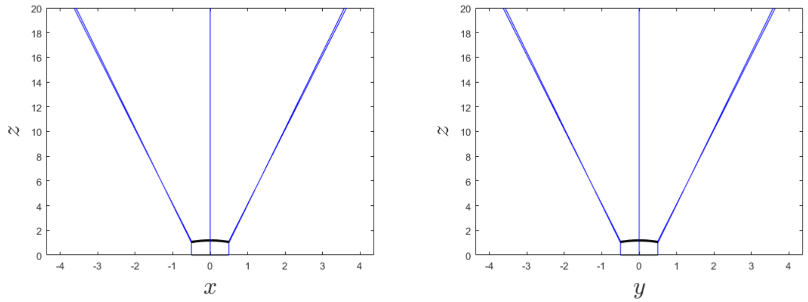


Elliptic (convex)

$u(x,y) = x^2 + y^2$

$\boldsymbol{P}$ positive definite

Eigenvalues $1, 1$

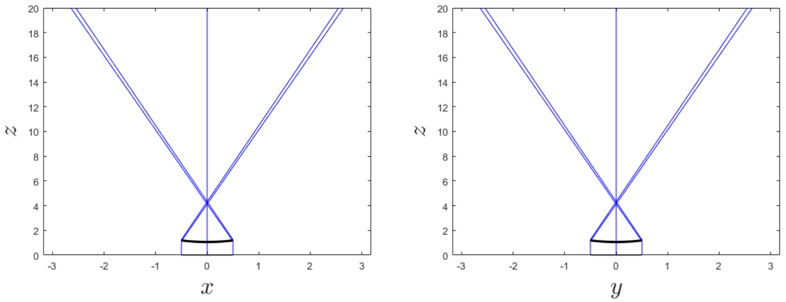


Elliptic (concave)

$u(x,y) = -x^2 - y^2$

$\boldsymbol{P}$ negative definite

Eigenvalues $-1, -1$

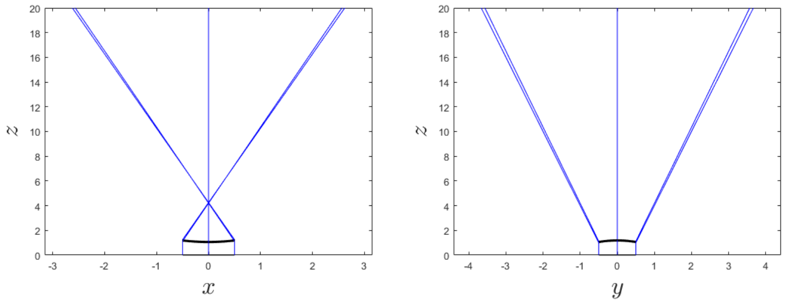


Hyperbolic

$u(x,y) = x^2 - y^2$

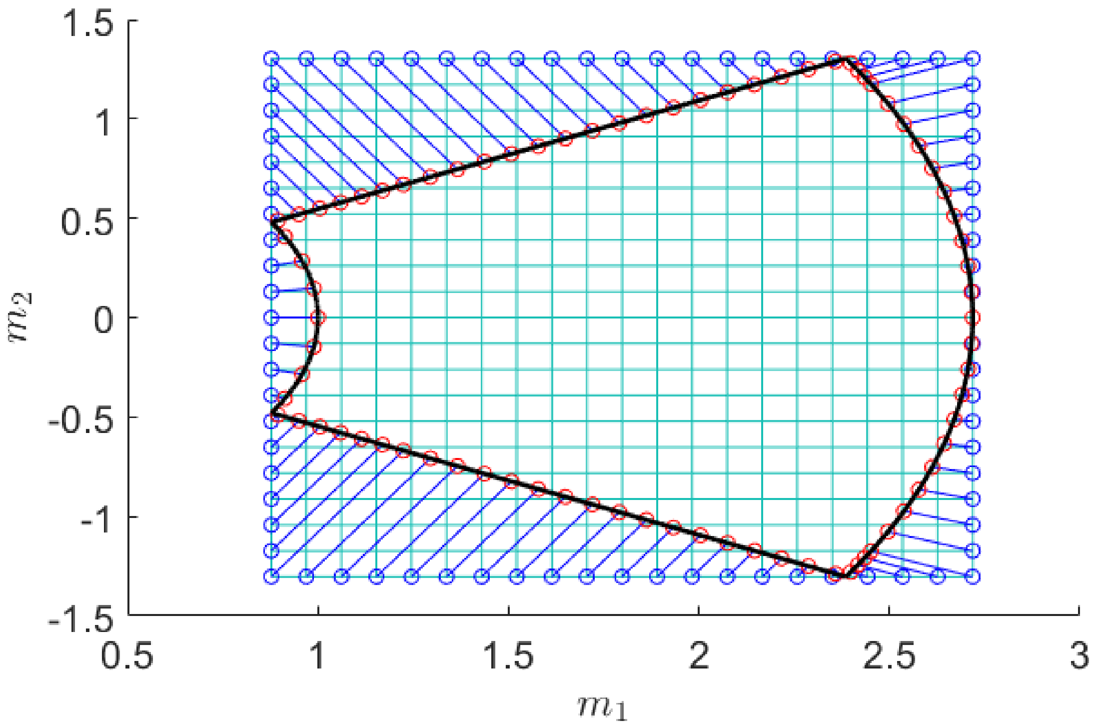
$\boldsymbol{P}$ indefinite

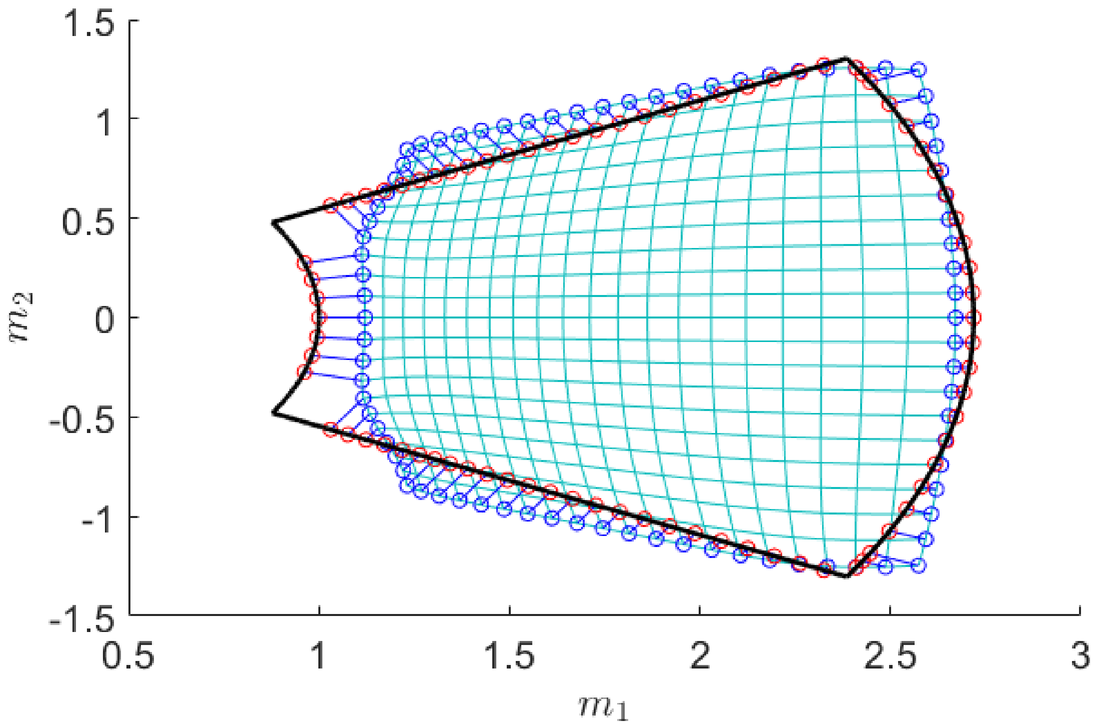
Eigenvalues $-1, 1$



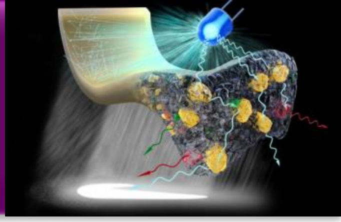
Numerical results

$X = [0,1] \times \left[-\frac{1}{2}, \frac{1}{2}\right]$, source intensity $F(x,y) = e^x$, uniform target intensity $G(x,y) = 1$





Numerical results



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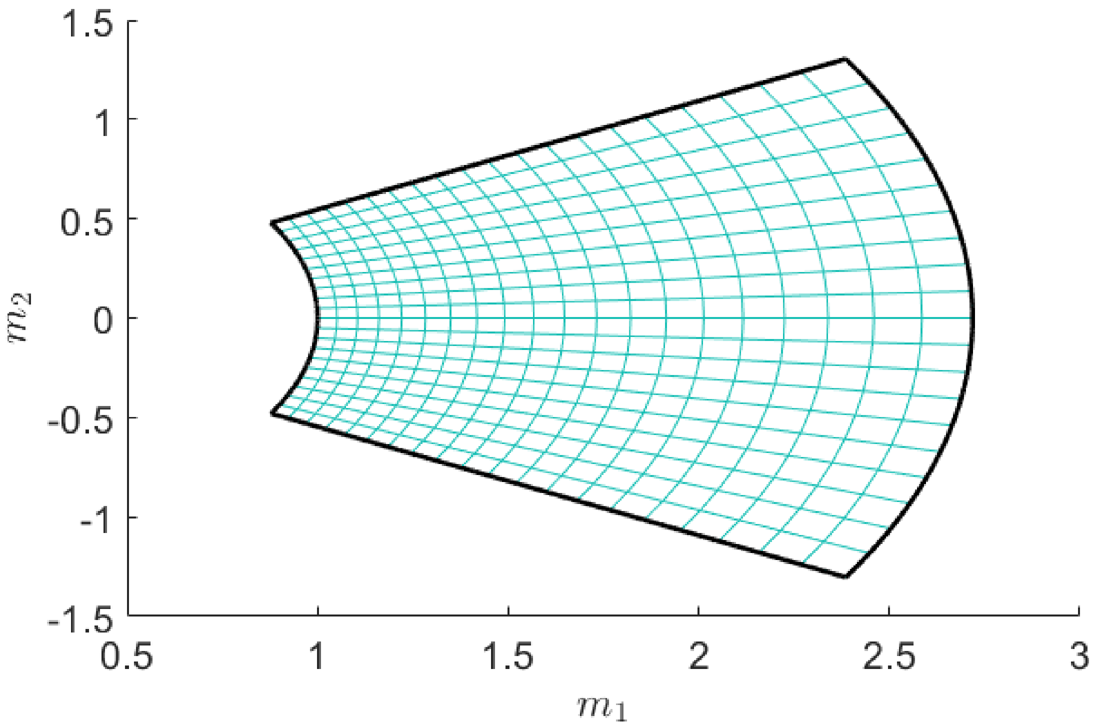

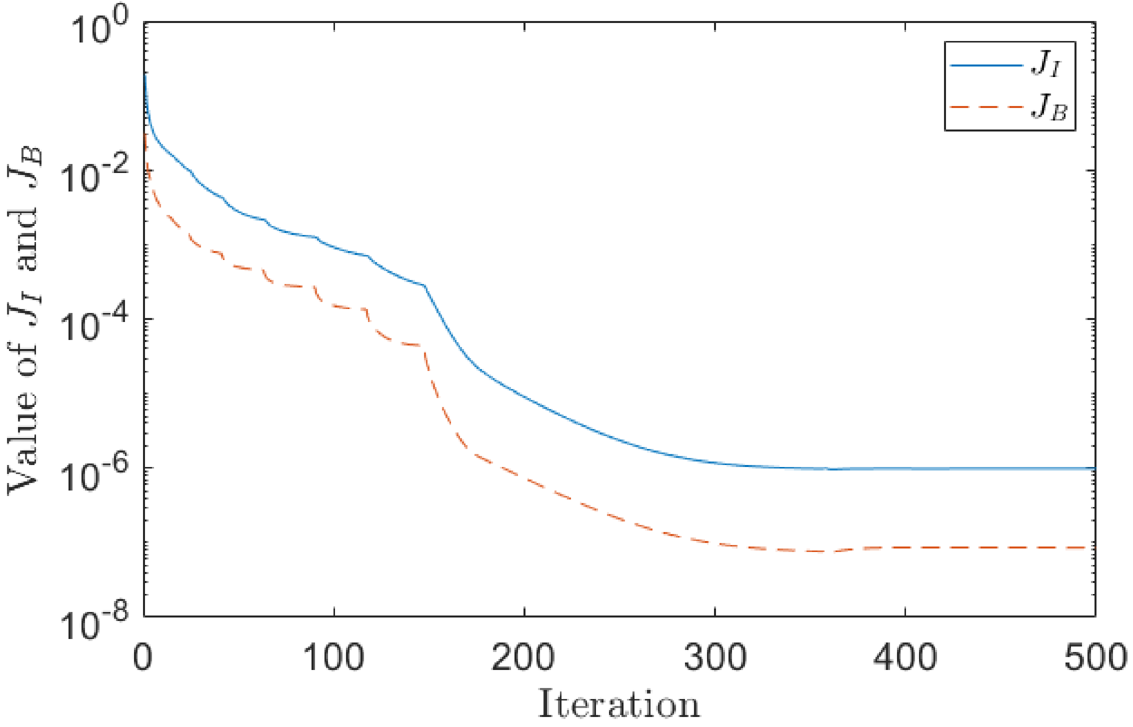

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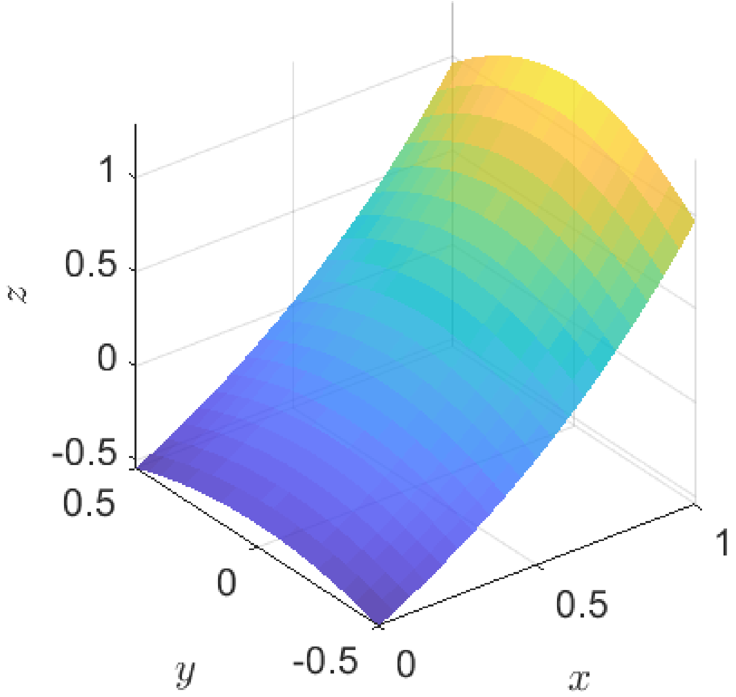

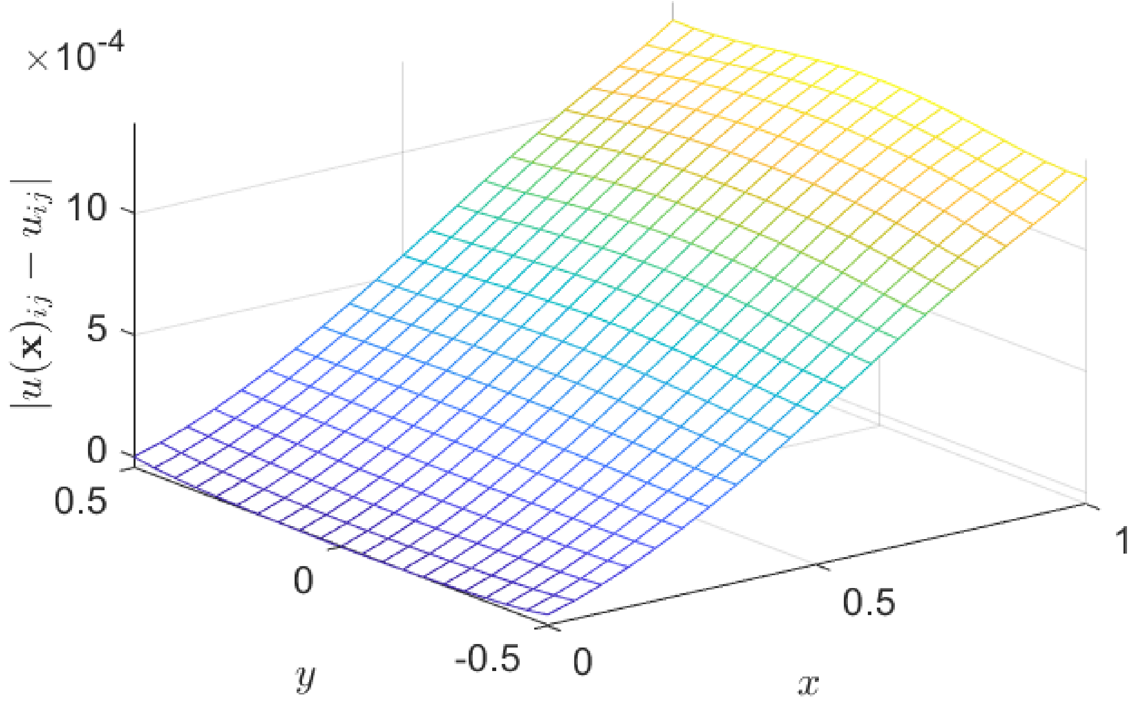

Numerical results

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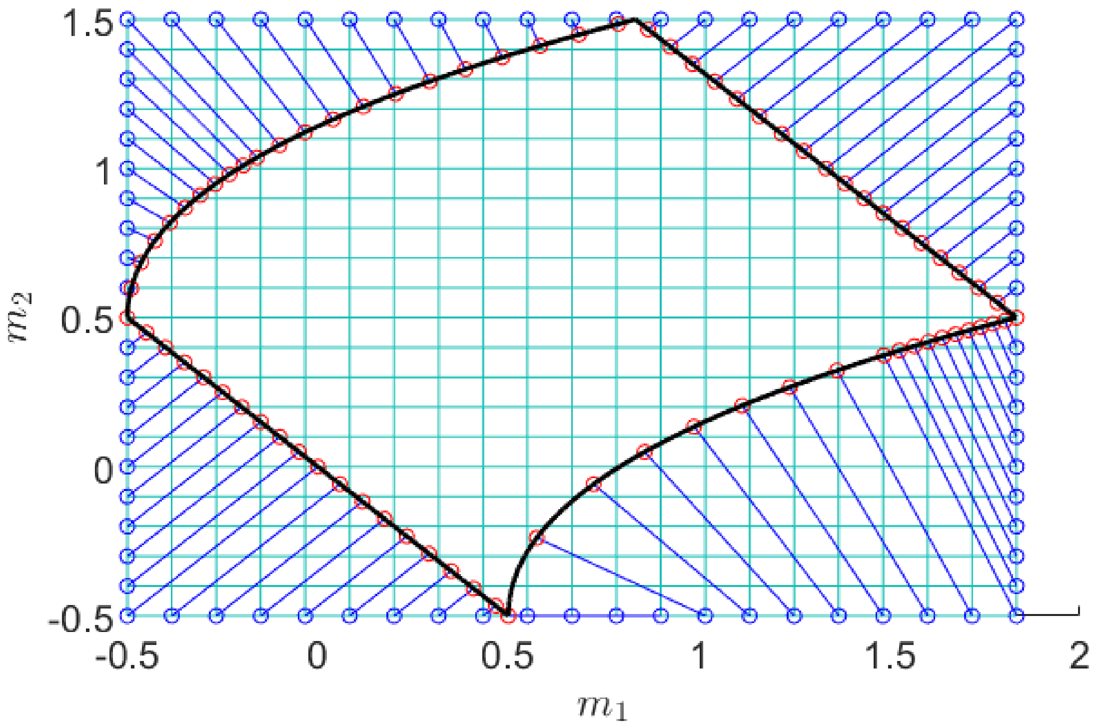

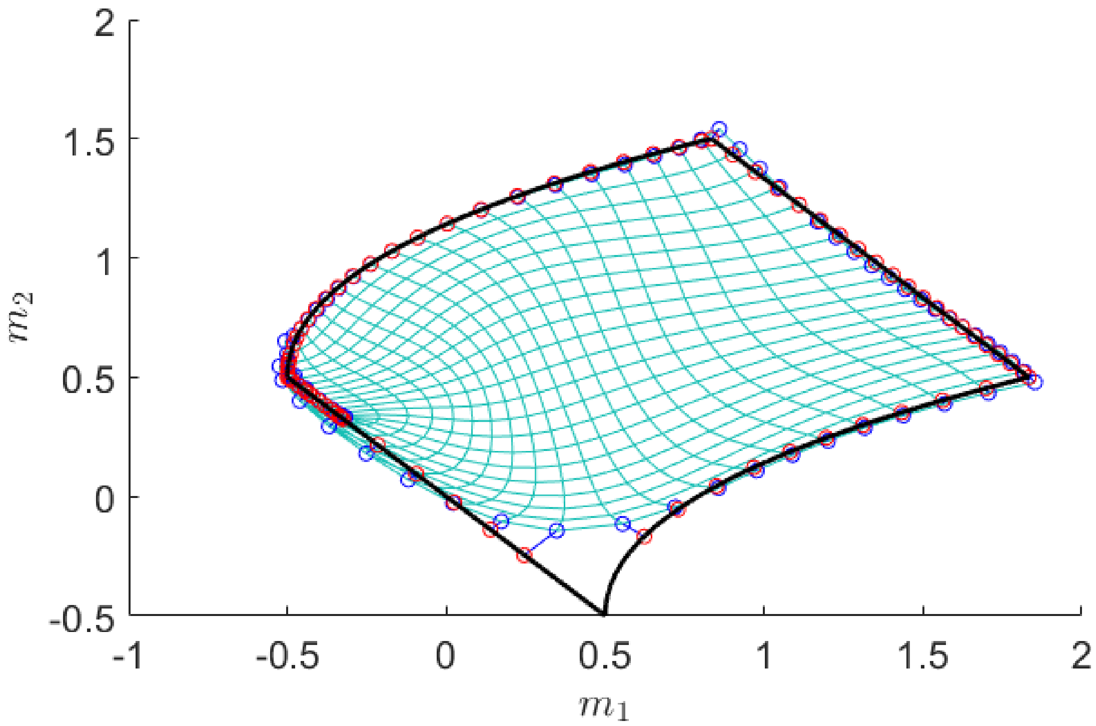

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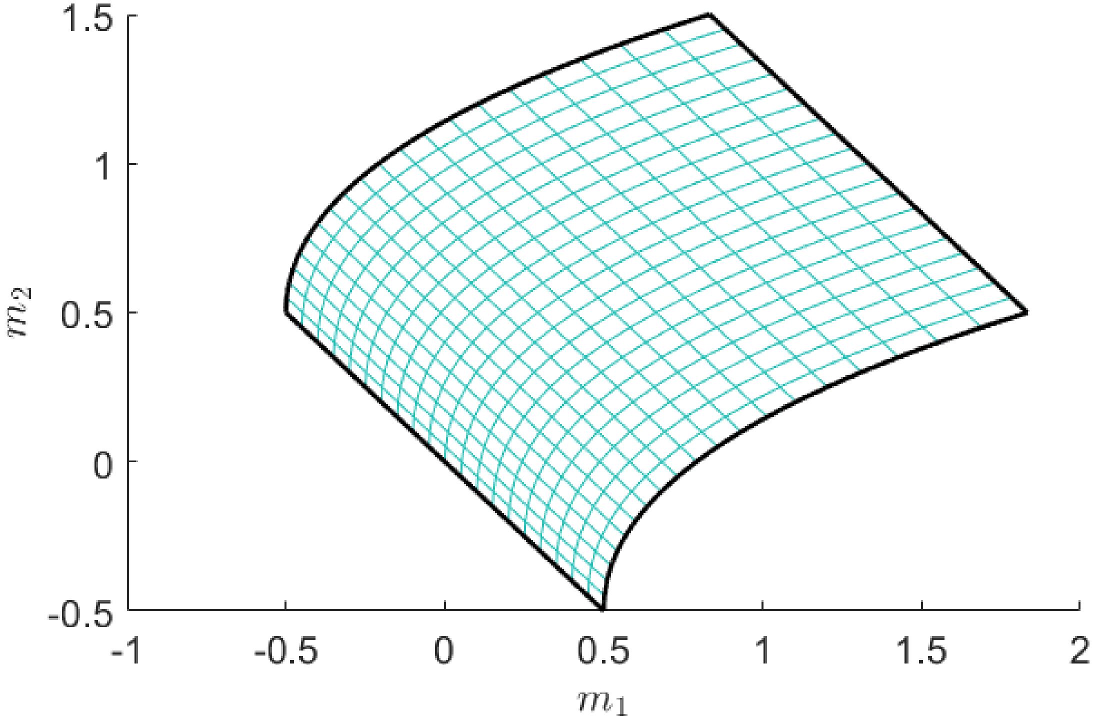
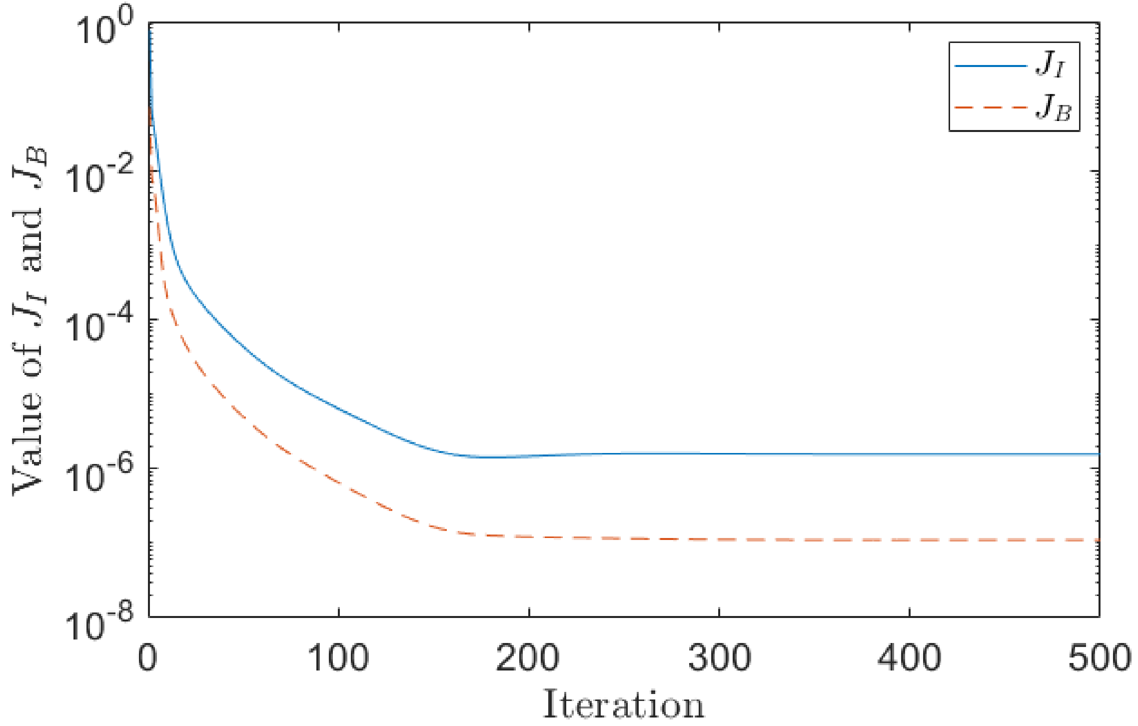
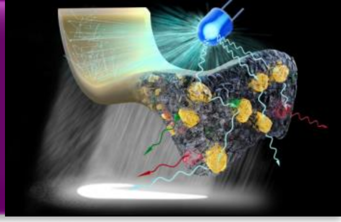




Conclusion



- A true difference between elliptical and hyperbolic surfaces
- Least-square solver works for some hyperbolic examples
- Need for a better boundary optimization method

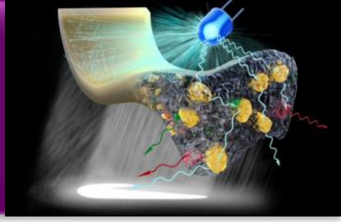
Alex Heemels – Workpackage B3

Multi-source Light Shaping

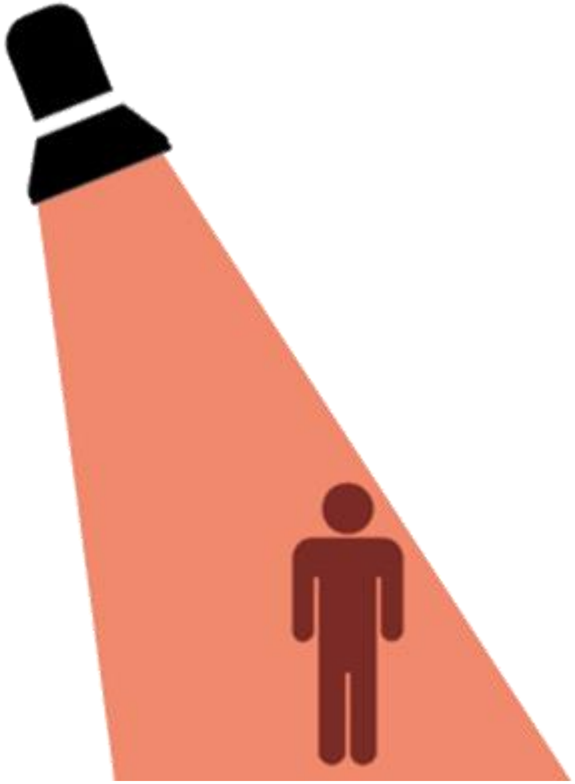
Alexander Heemels

TU Delft

Why Multi-Source Systems?



- Higher light throughput



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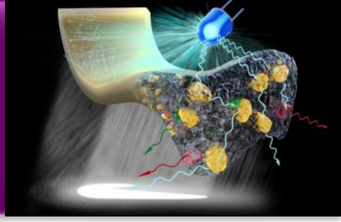
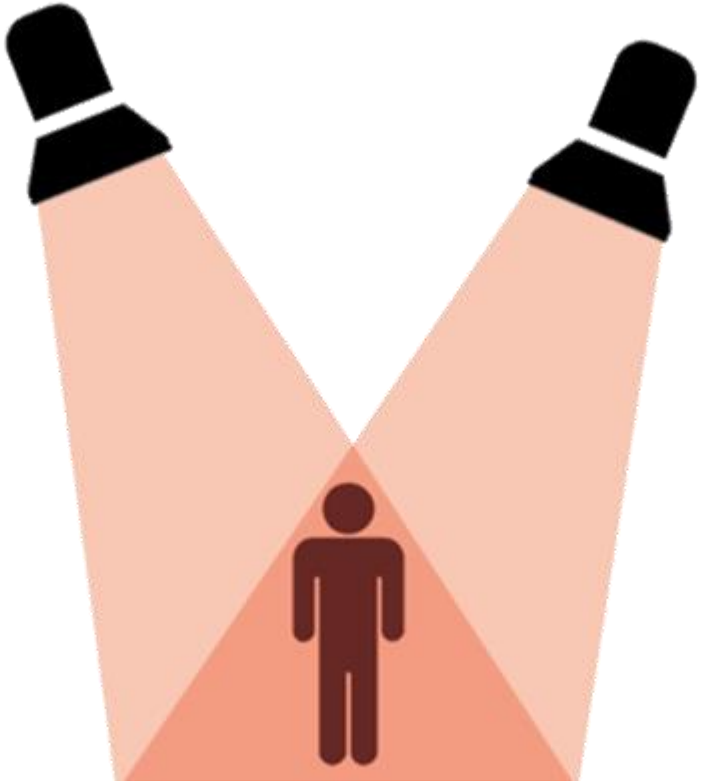


- Higher light throughput

Why Multi-Source Systems?

- Higher light throughput
- Increase system lifetime



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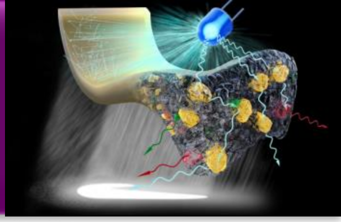
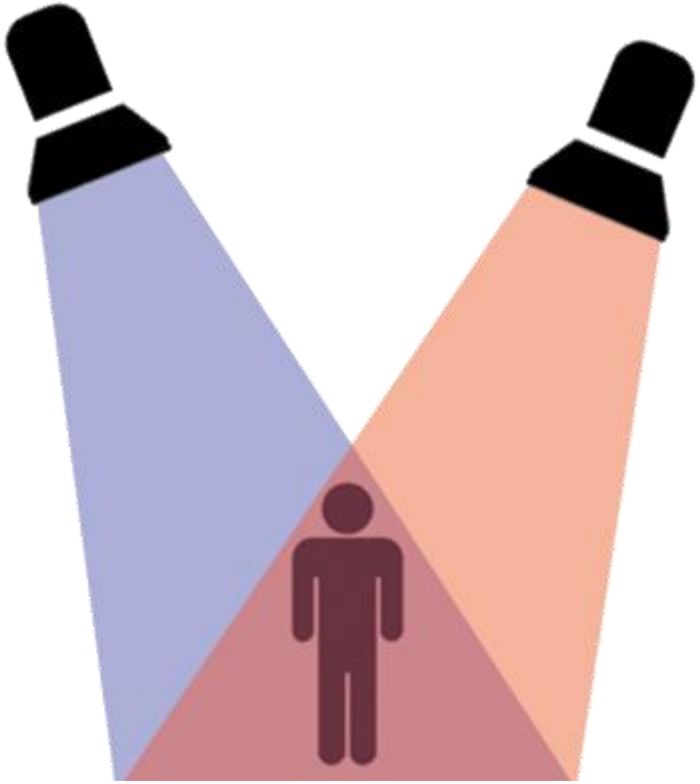


- Higher light throughput
- Increase system lifetime
- Colour mixing



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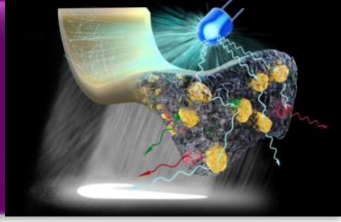

https://www.signify.com/en-gb/our-company/news/press-release-archive/2016/20160310-philips-lighting-launches-new-led-family-of-street-lights-futureproofed-by-design-for-Internet-of-things-age

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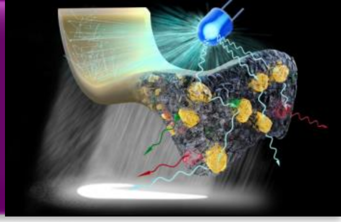
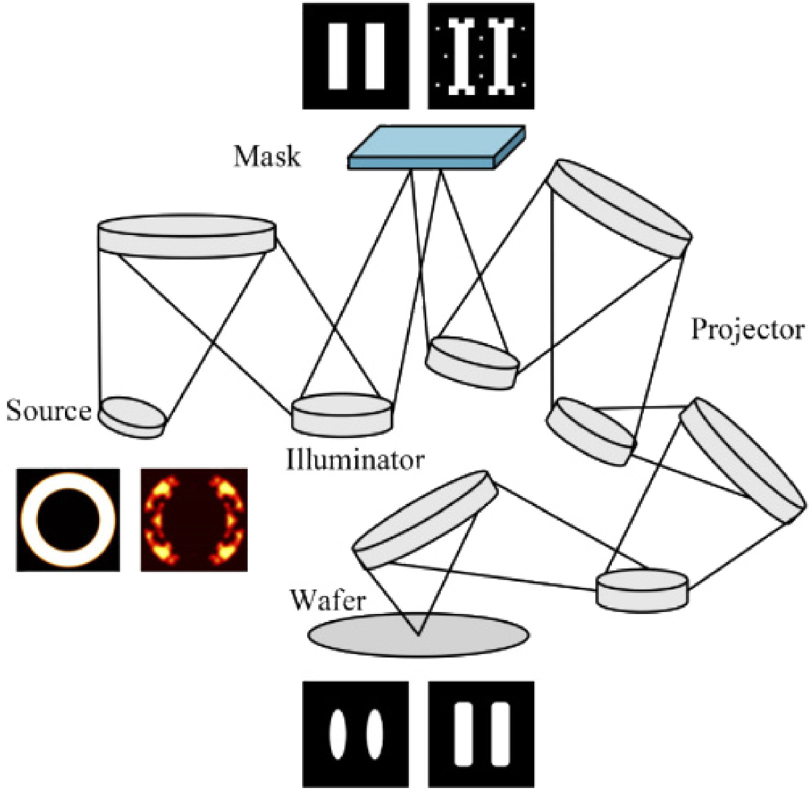
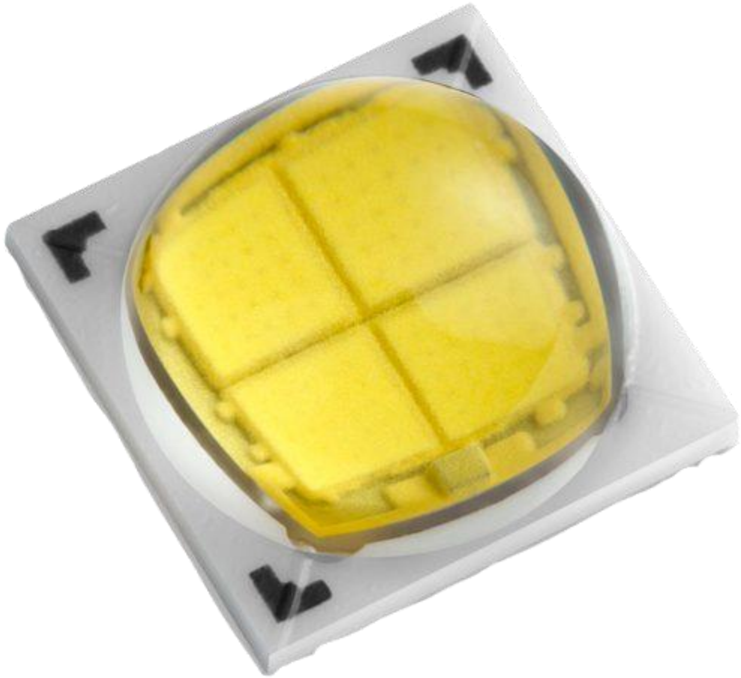




https://www.signify.com/en-gb/our-company/news/press-release-archive/2016/20160310-philips-lighting-launches-new-led-family-of-street-lights-futureproofed-by-design-for-Internet-of-things-age

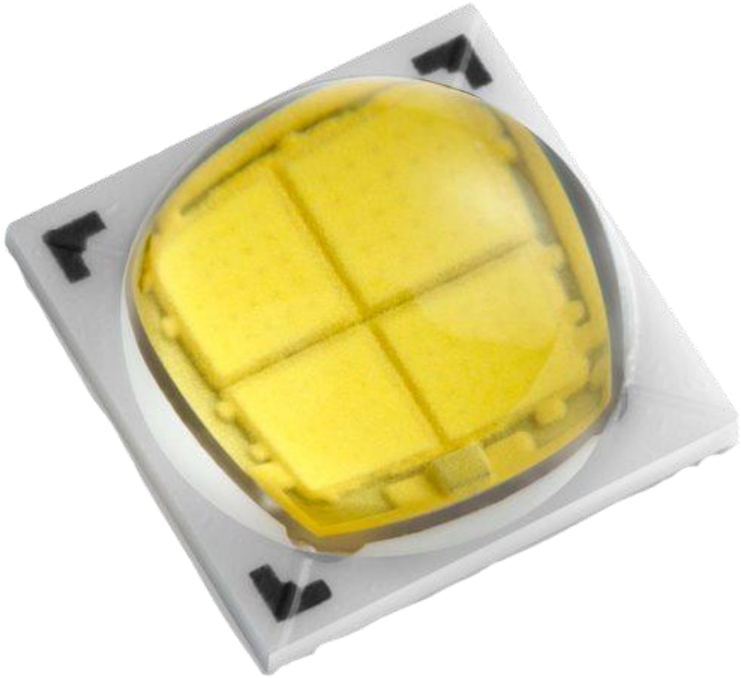
Zinan Zhang, Sikun Li, Xiangzhao Wang, Wei Cheng, and Yuejing Qi, "Source mask optimization for extreme-ultraviolet lithography based on thick mask model and social learning particle swarm optimization algorithm," Opt. Express **29**, 5448-5465 (2021)

Why Multi-Source Systems?



https://lumileds.com/products/high-power-leds/luxeon-m/

Why Multi-Source Systems?



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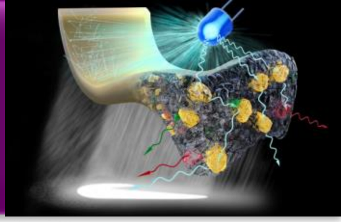

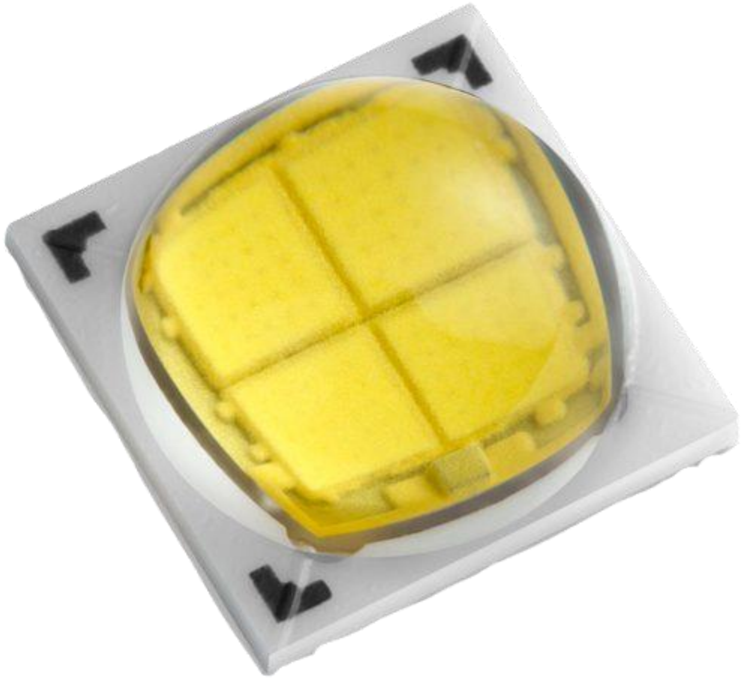

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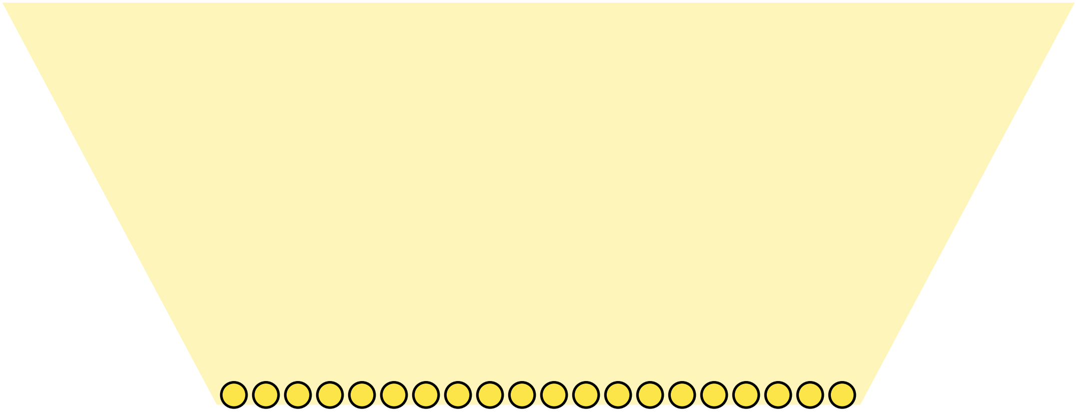
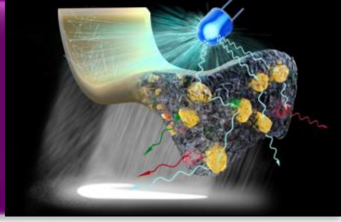
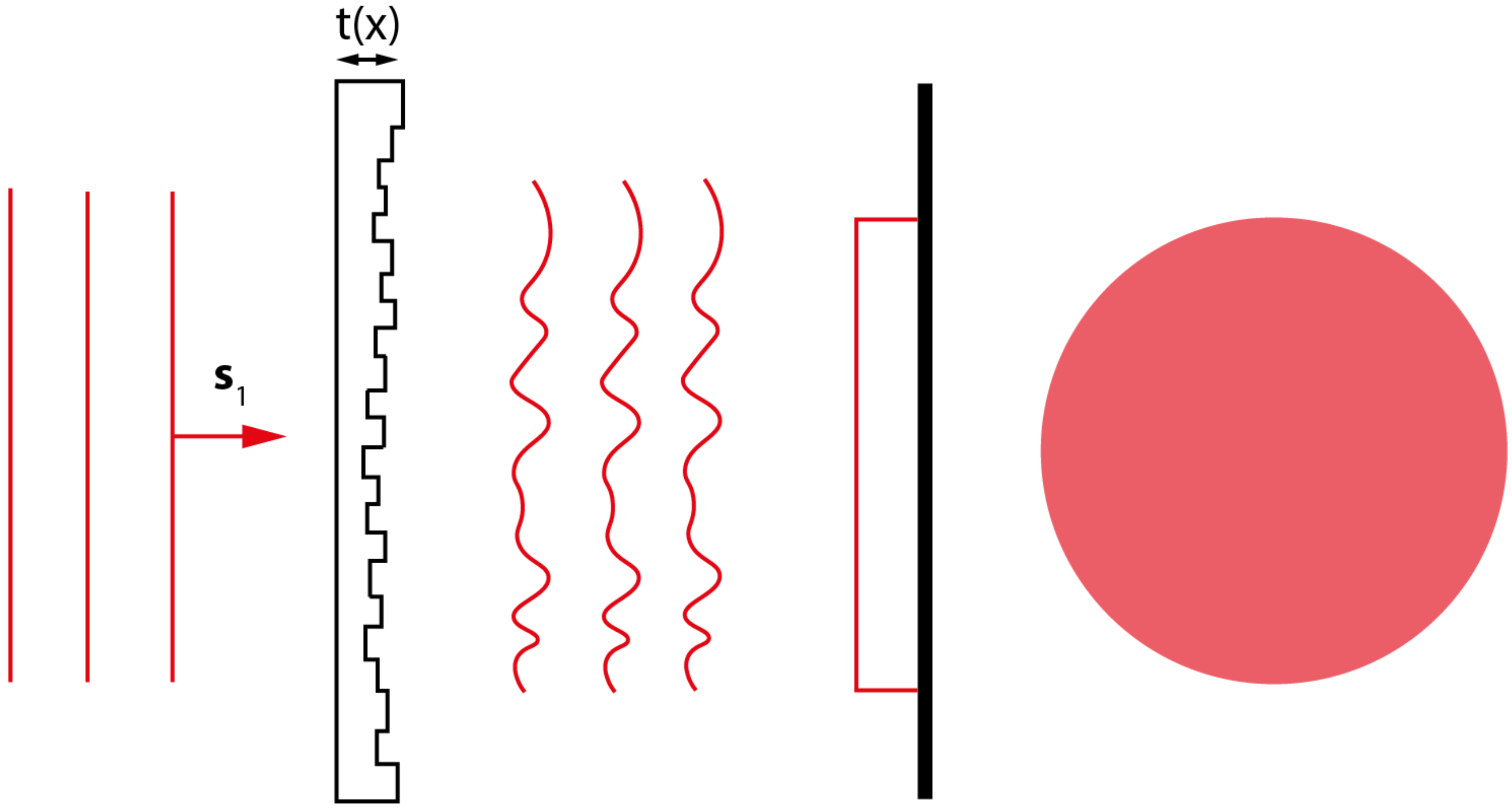


Physical Background: Diffractive Optics





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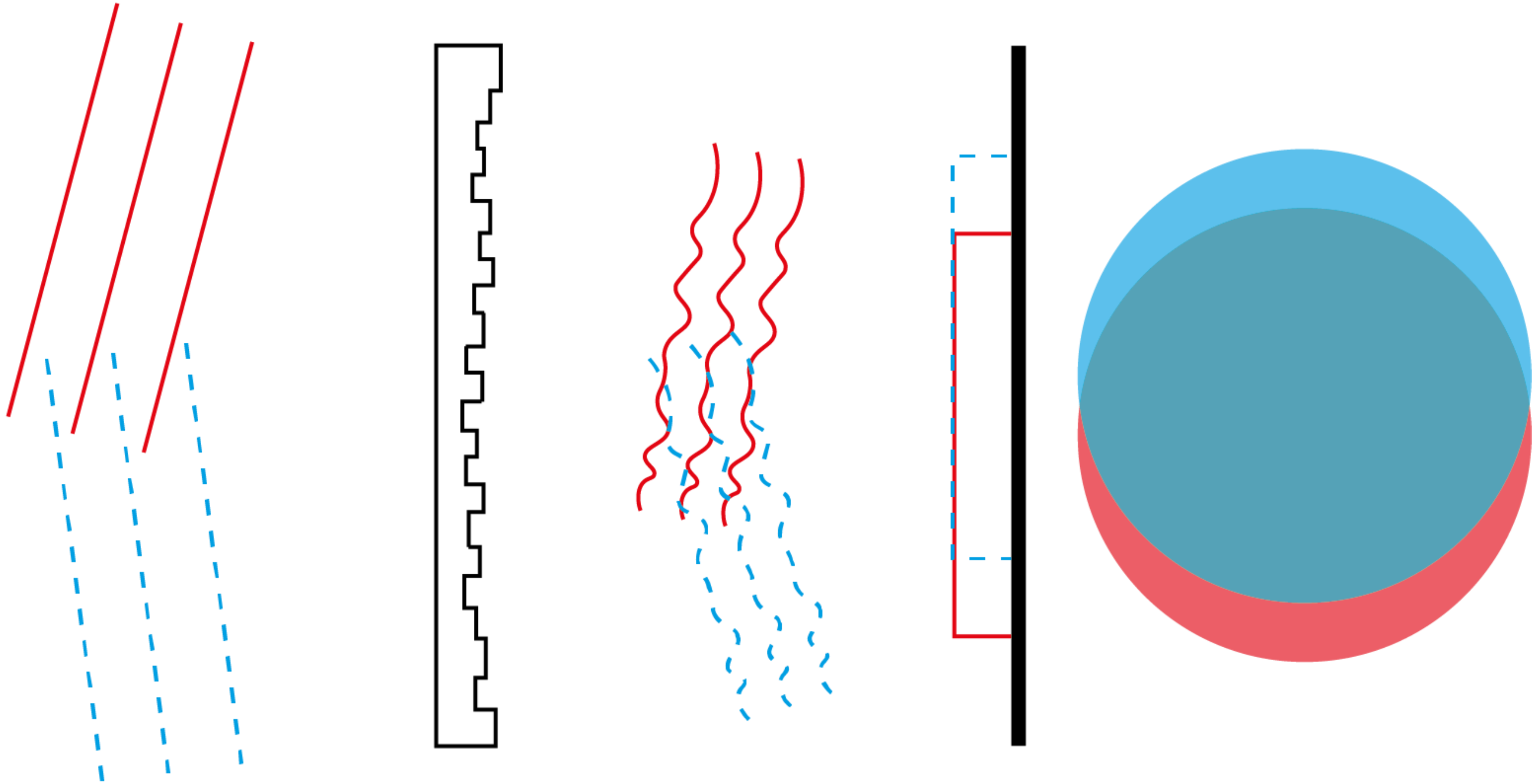

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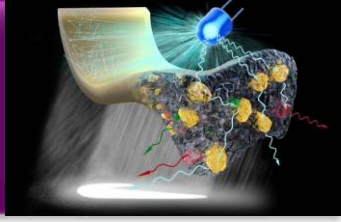
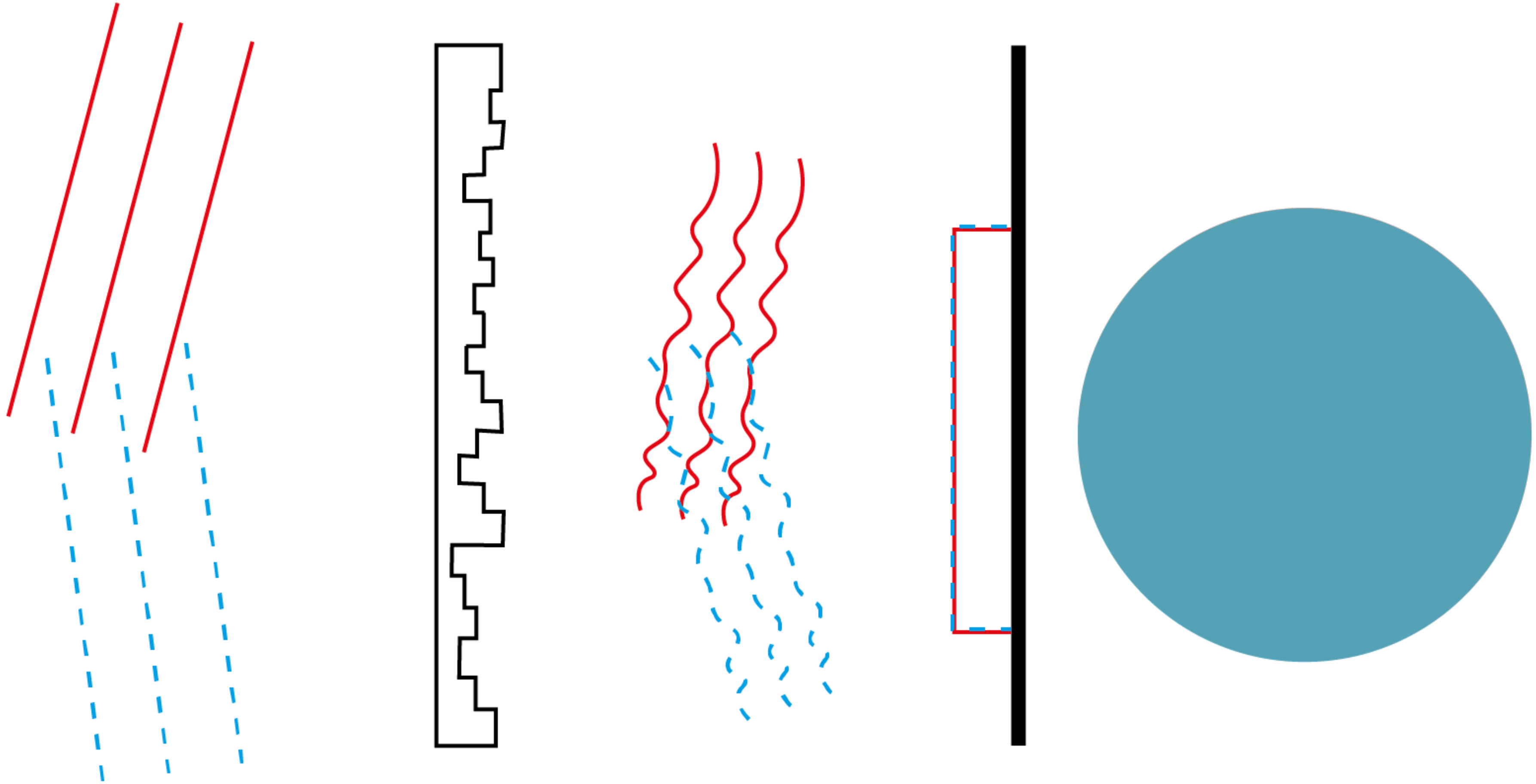
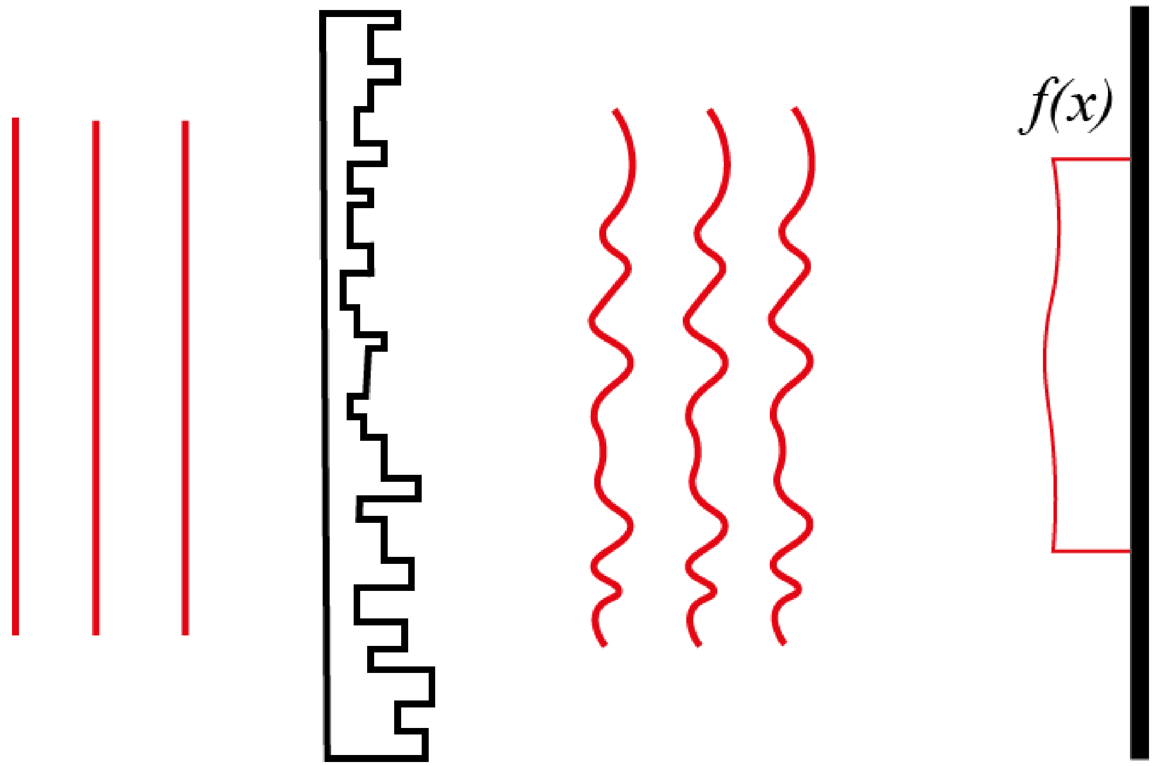




Problem Definition

Usually DoE is designed using a source on the optical axis for a desired distribution $f(\mathbf{x})$



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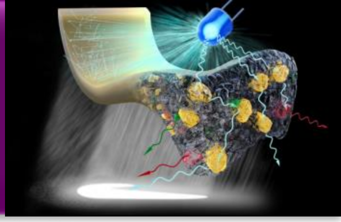
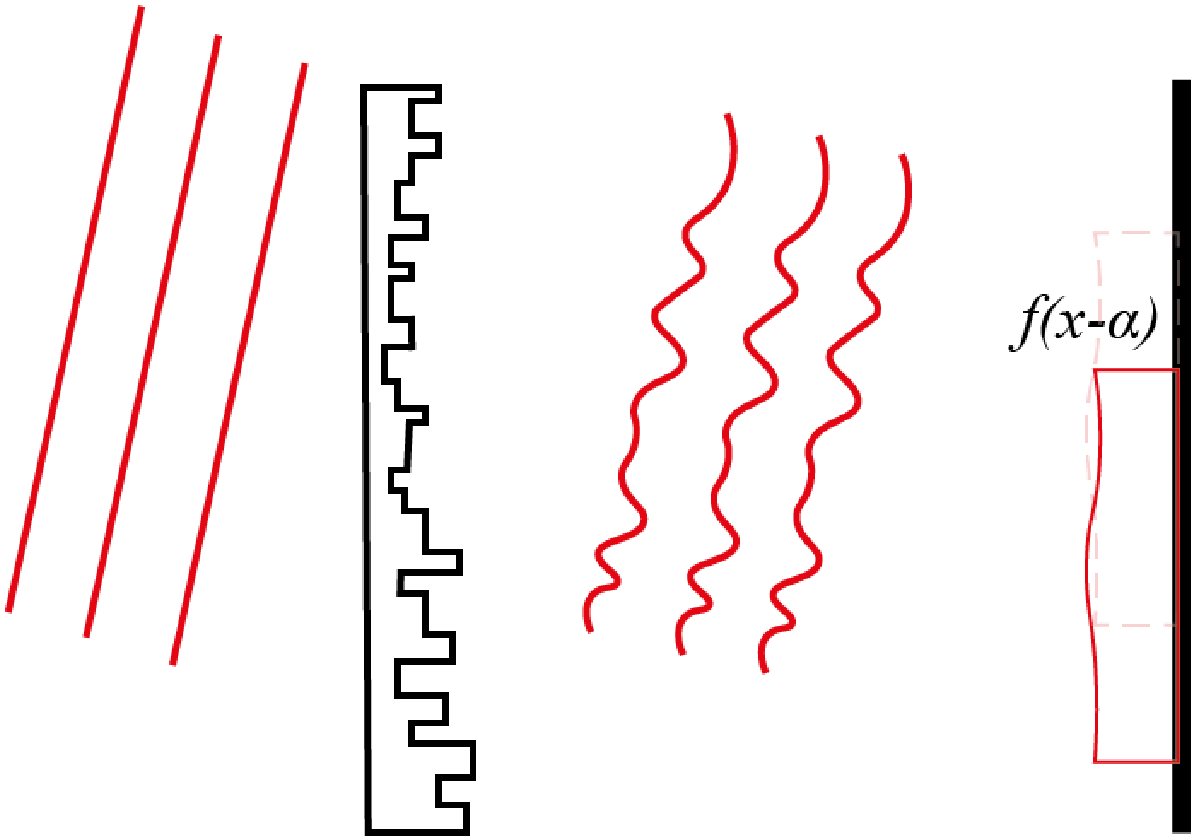
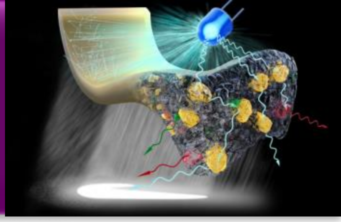


Usually DoE is designed using a source on the optical axis for a desired distribution $f(\mathbf{x})$

Changing incidence angle of plane wave spatially shift the illumination distribution.

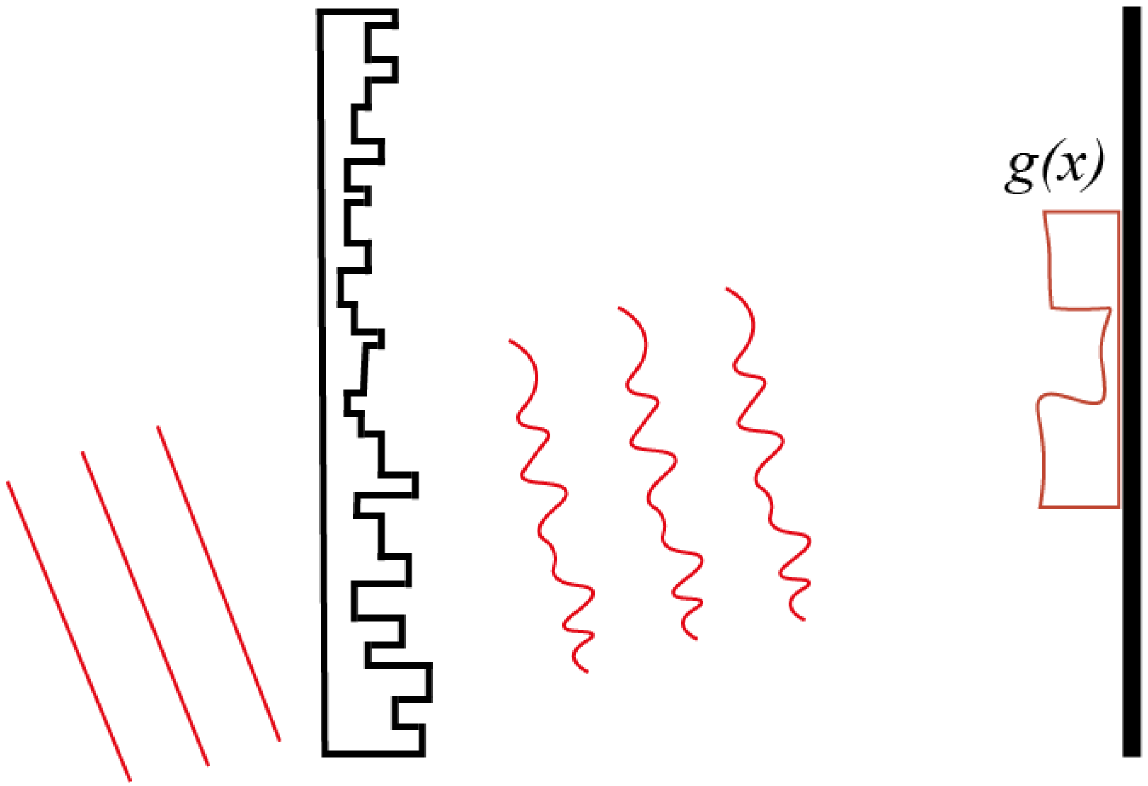


Problem Definition



Changing incidence angle of plane wave spatially shift the illumination distribution.

Can we find a distribution $g(\mathbf{x})$?



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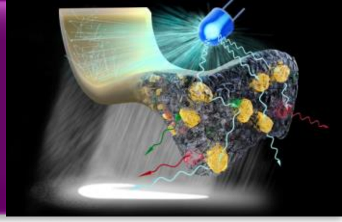
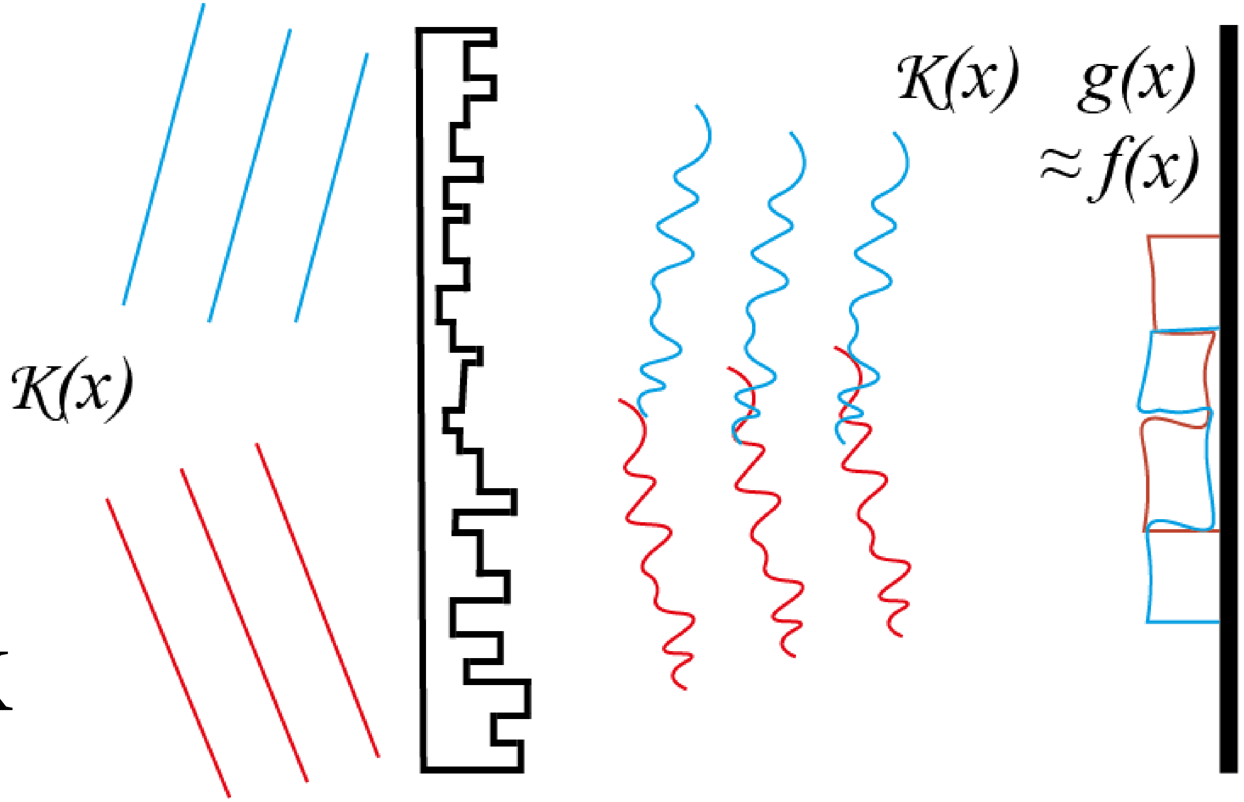
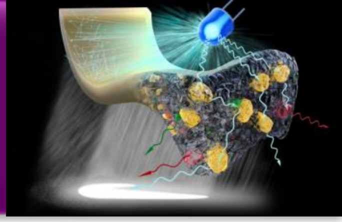


Changing incidence angle of plane wave spatially shift the illumination distribution.

Can we find a distribution $g(\mathbf{x})$? Which for given incidence angles K approximates $f(\mathbf{x})$





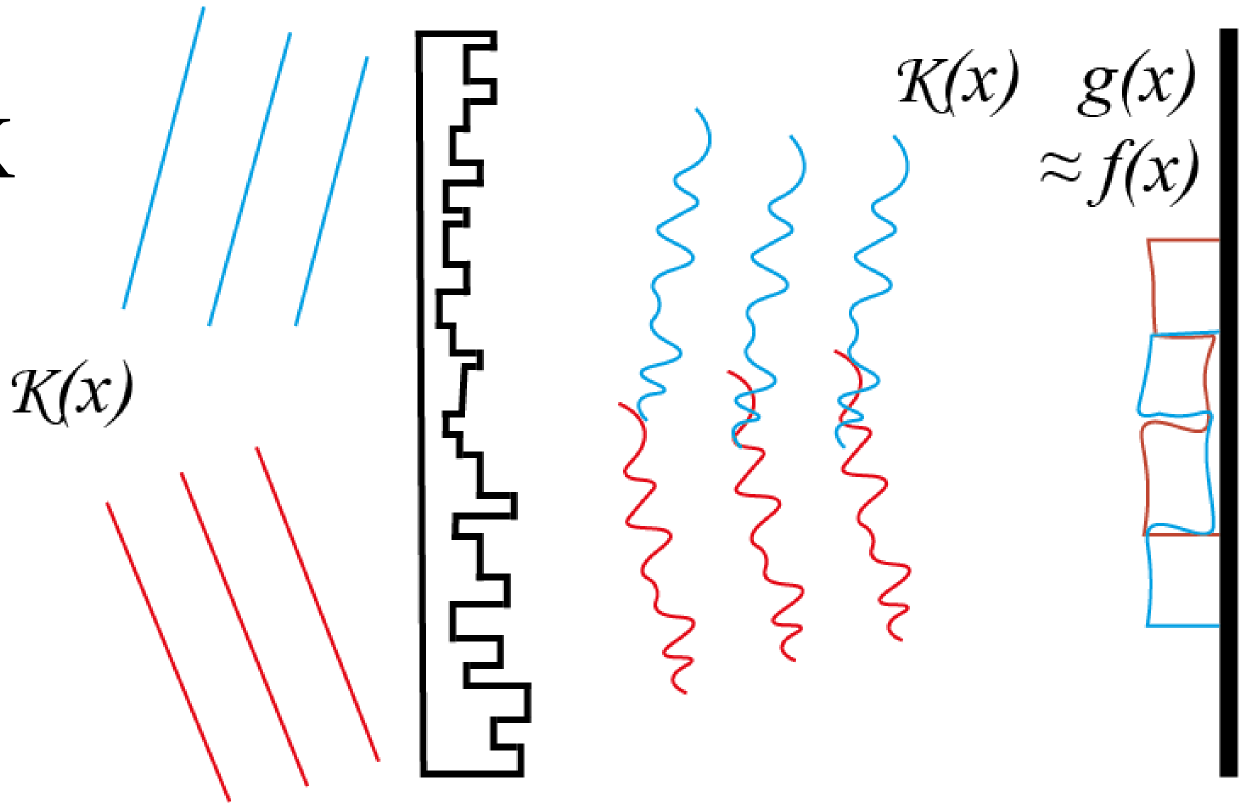
Problem Definition

Can we find a distribution $g(\mathbf{x})$?

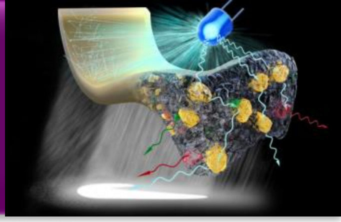
Which for given incidence angles K

approximates $f(\mathbf{x})$

$$\underbrace{f(\mathbf{x})}_{\text{DESIRED ILLUMINATION}} = \int_{-\infty}^{+\infty} \underbrace{g(\xi)}_{\text{INTERMEDIATE ILLUMINATION}} \underbrace{\mathcal{K}(\mathbf{x}-\xi)}_{\text{ILLUMINATION KERNEL}} \mathrm{d}\xi$$



Solving the Deconvolution



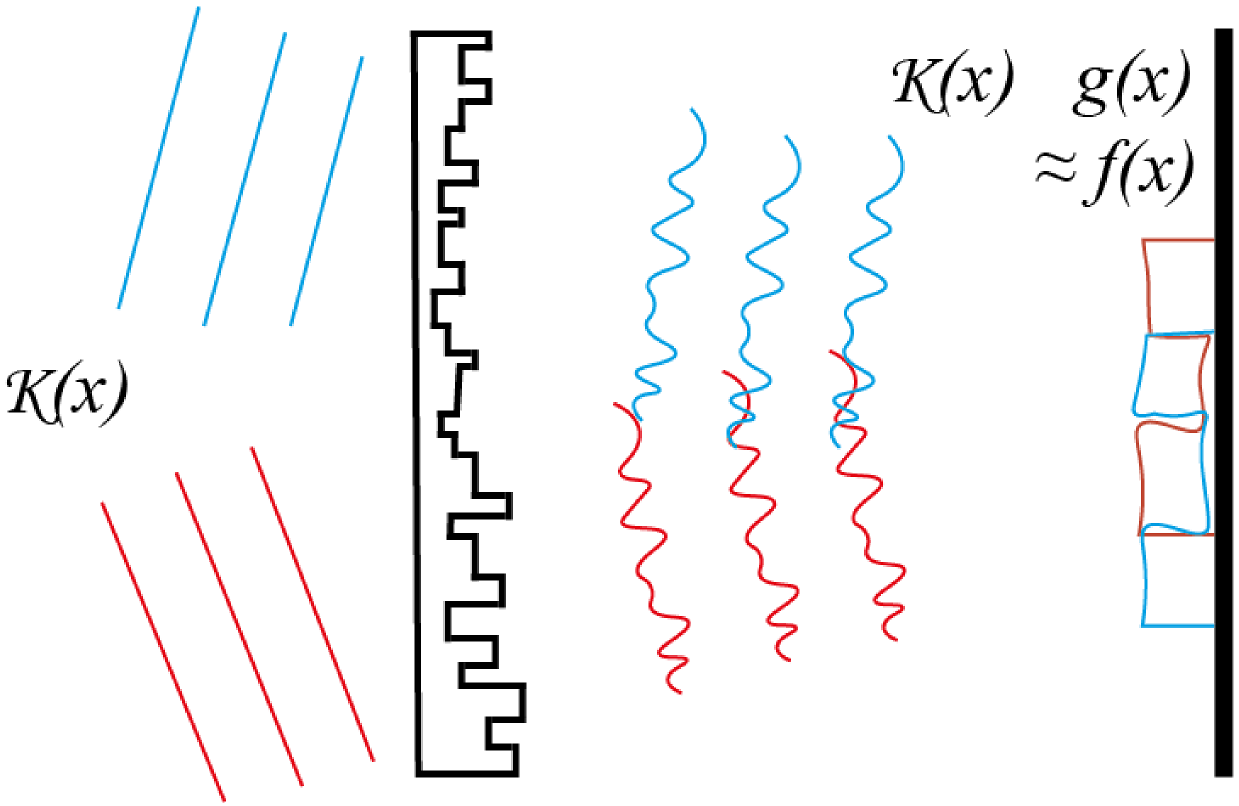
Start with original convolution

which approximates $f(x)$

$$\tilde{f}(\mathbf{x}) = \mathrm{K}(\mathbf{x}) \text{ å } g(\mathbf{x})$$

Discretize into vector convolution

$$\tilde{\mathbf{f}} = \mathbf{k} \text{ å } \mathbf{g}$$



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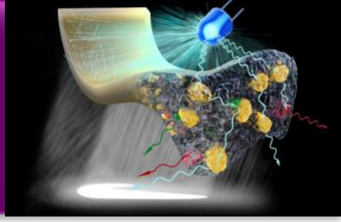
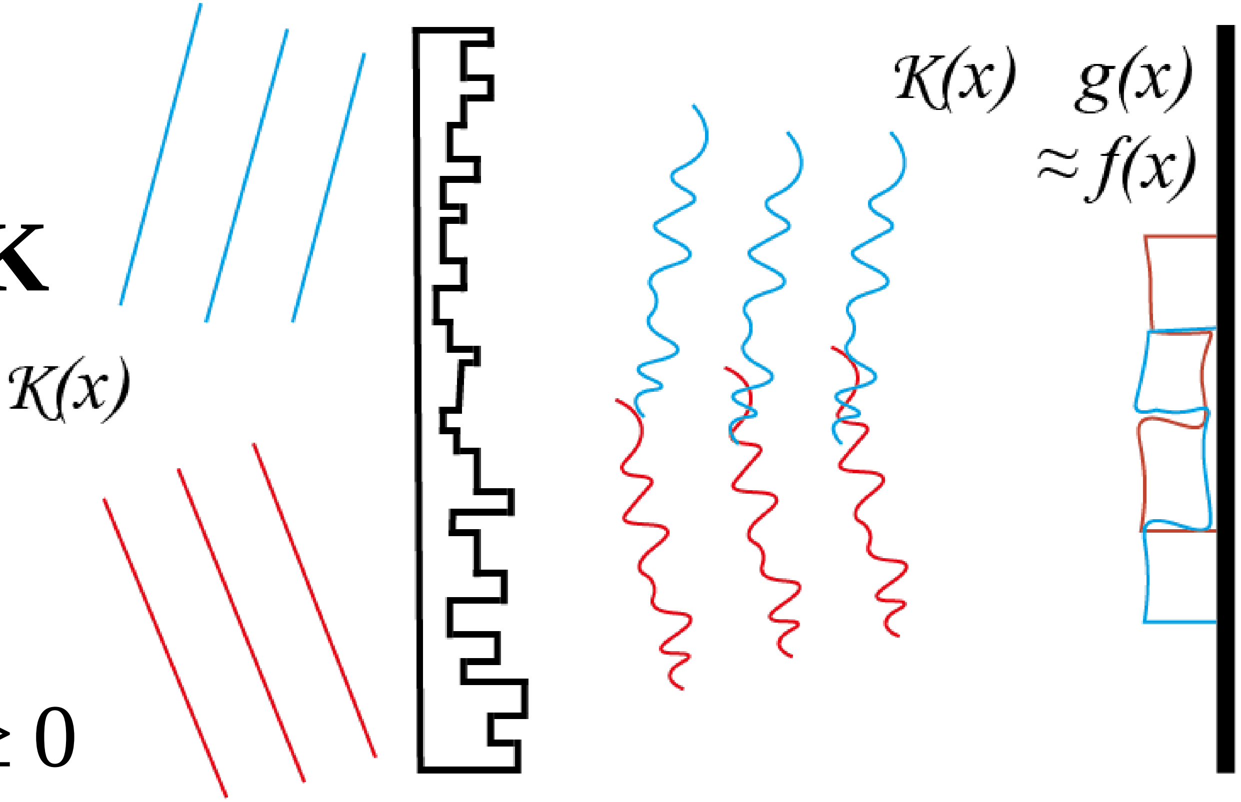


Convert vector convolution into vector matrix multiplication by converting $\mathbf{k}$ into a Toeplitz matrix $\mathbf{K}$

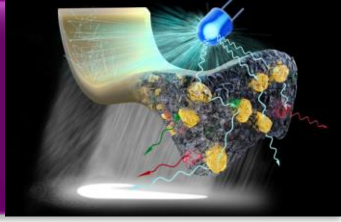
$$\tilde{\mathbf{f}} = \mathbf{K}\mathbf{g}$$

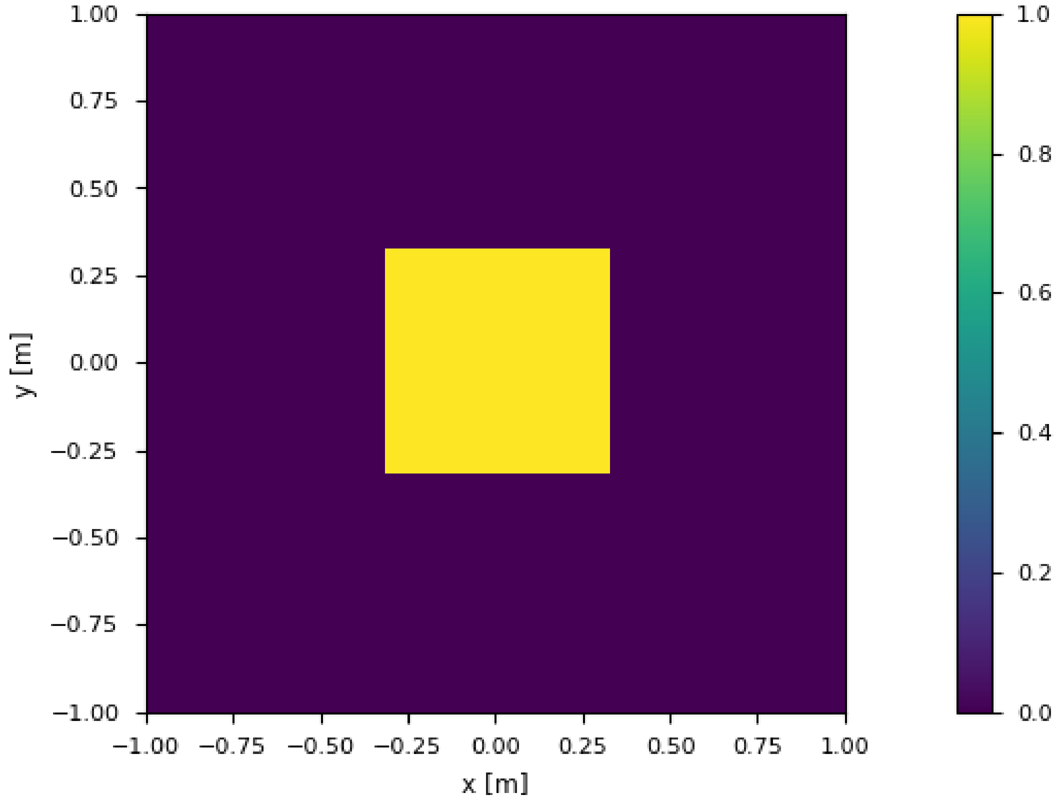
Solve the linear problem:

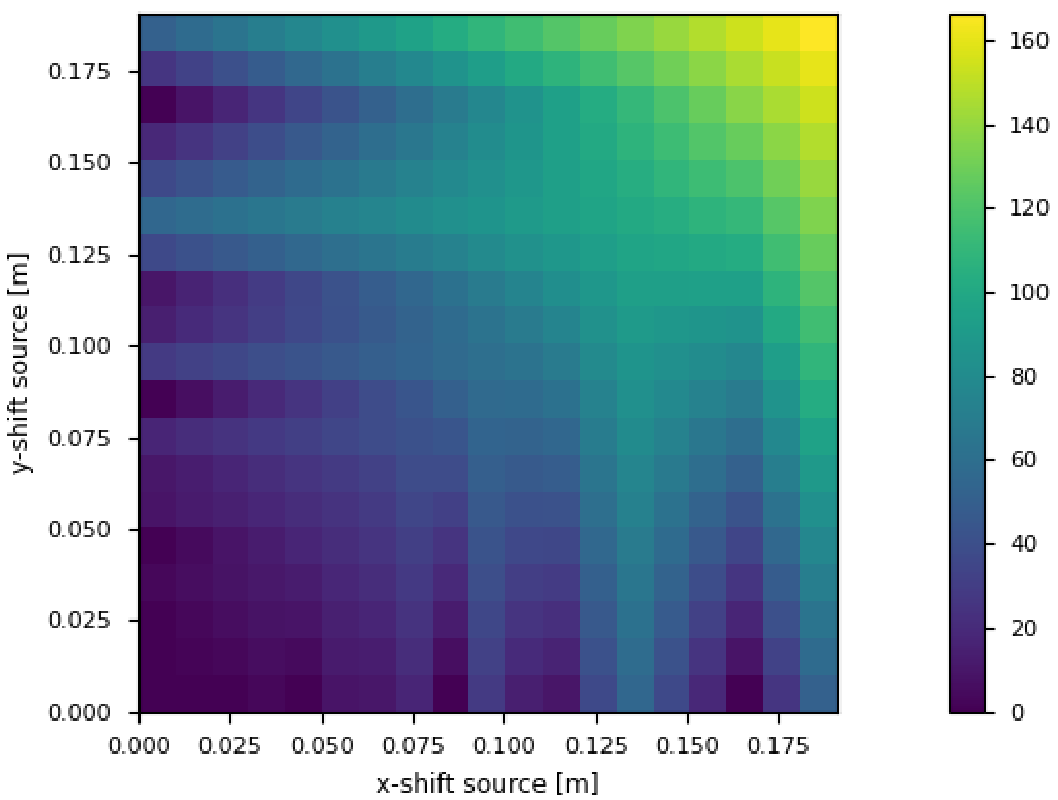
$$\min_{\mathbf{g}} \|\mathbf{f} - \mathbf{K}\mathbf{g}\|_2 \quad \text{subjected to} \quad \mathbf{g} \geq 0$$



Results 2 Sources







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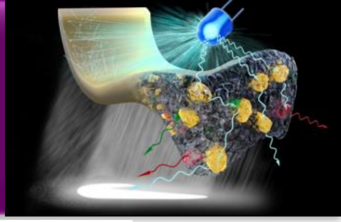

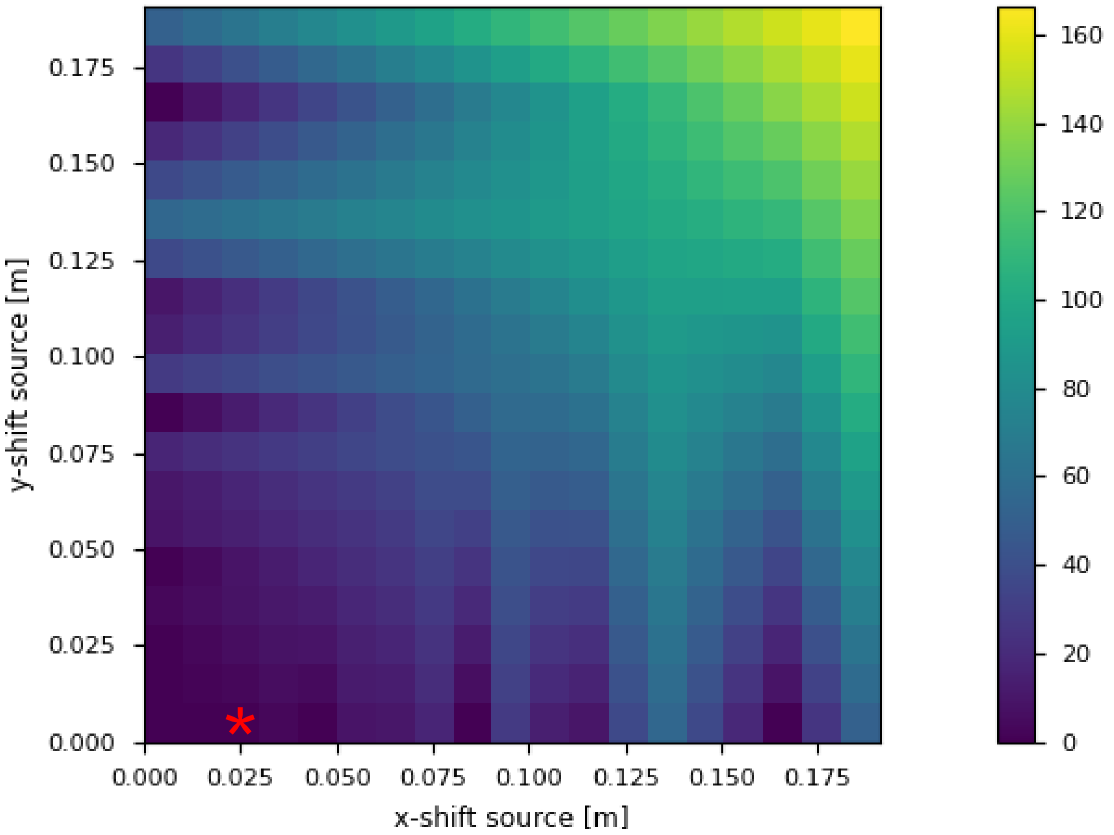

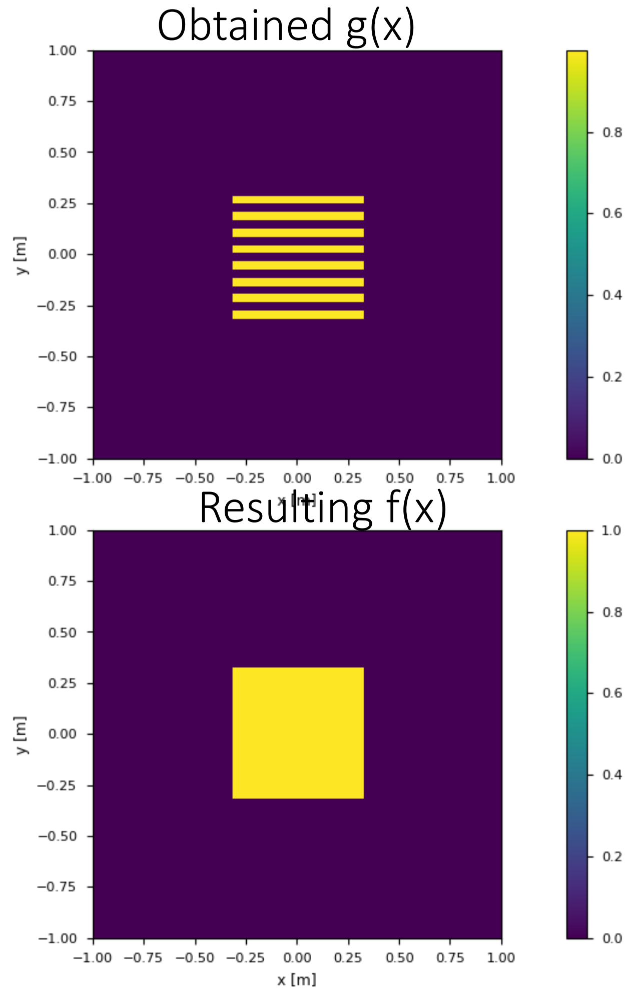

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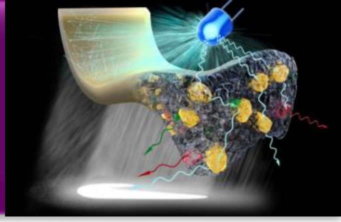
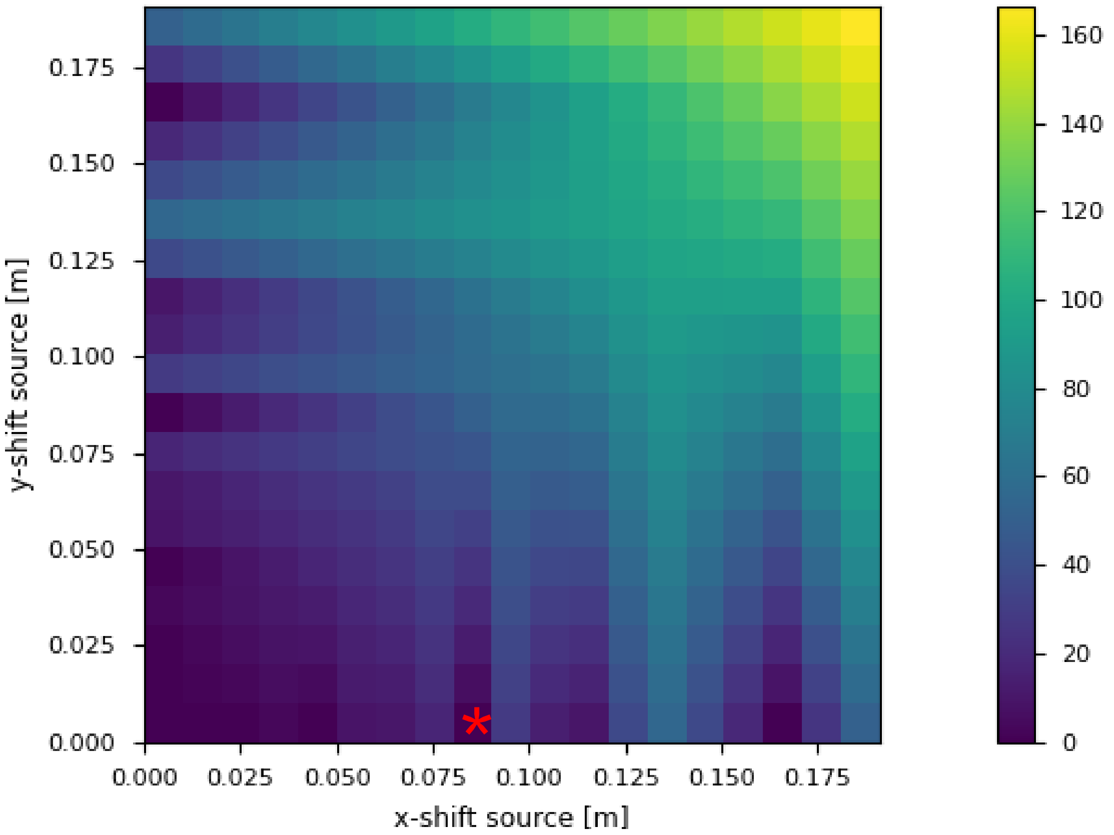
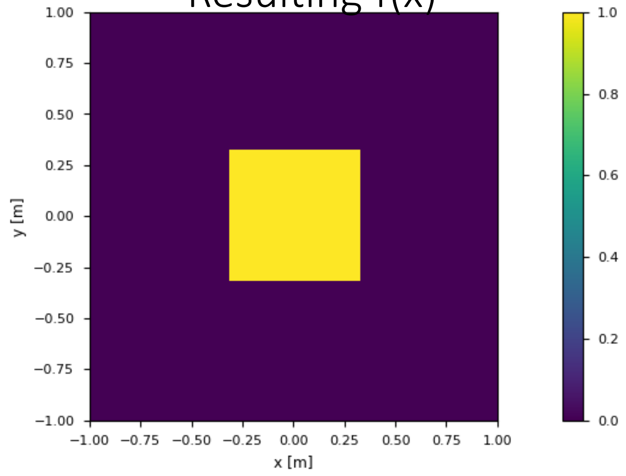




Obtained g(x)

Resulting f(x)



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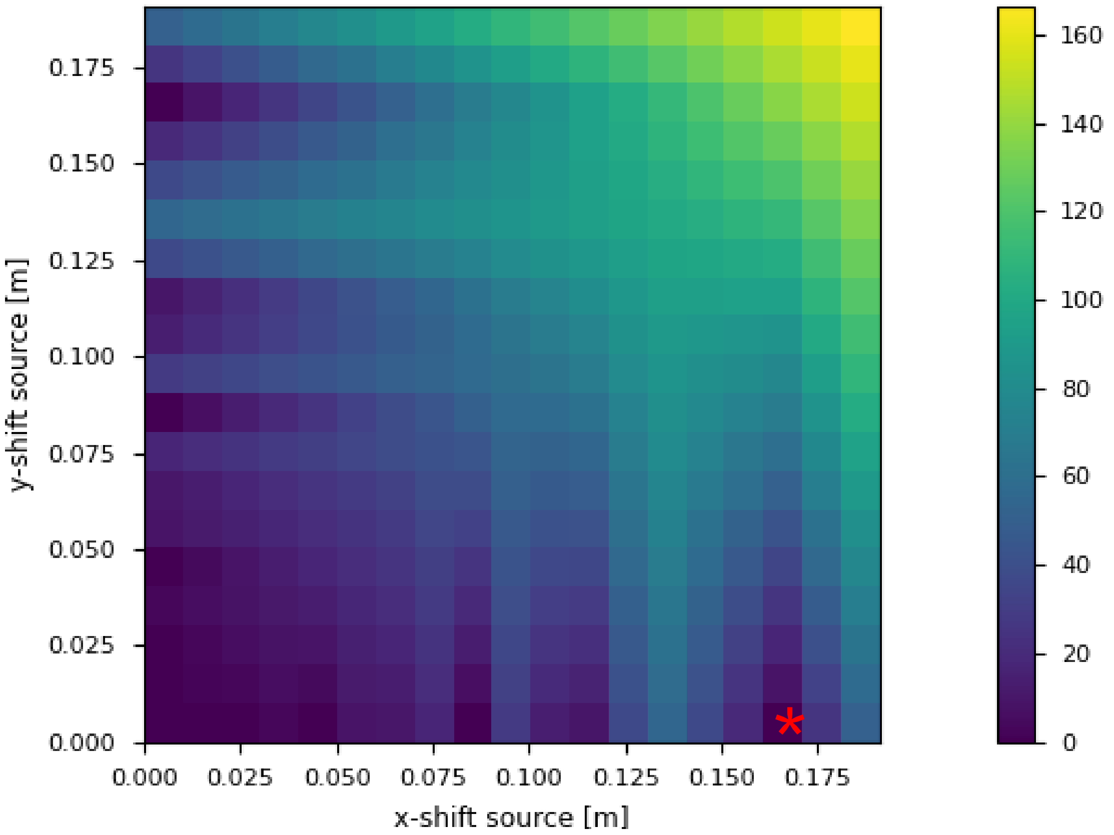

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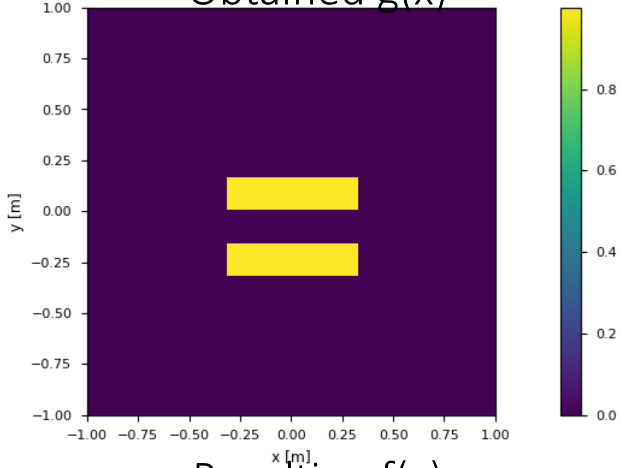

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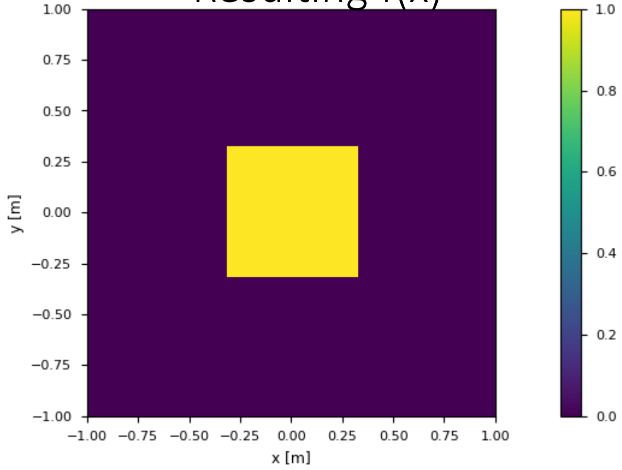

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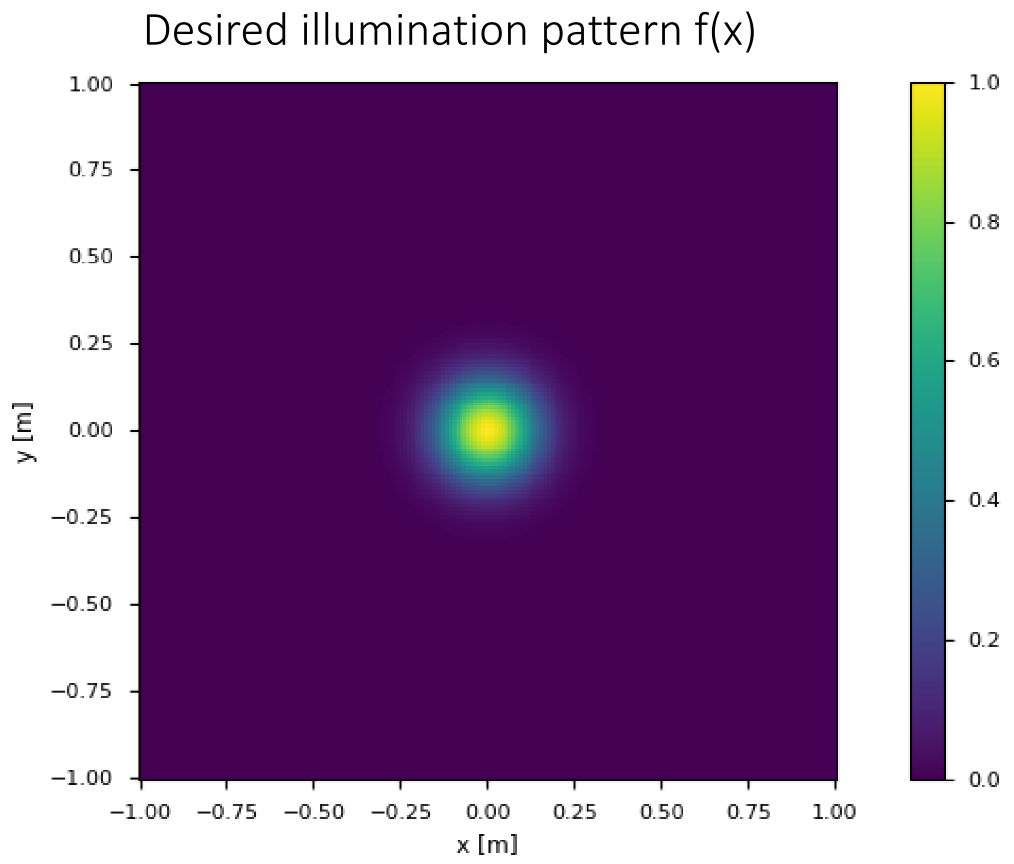

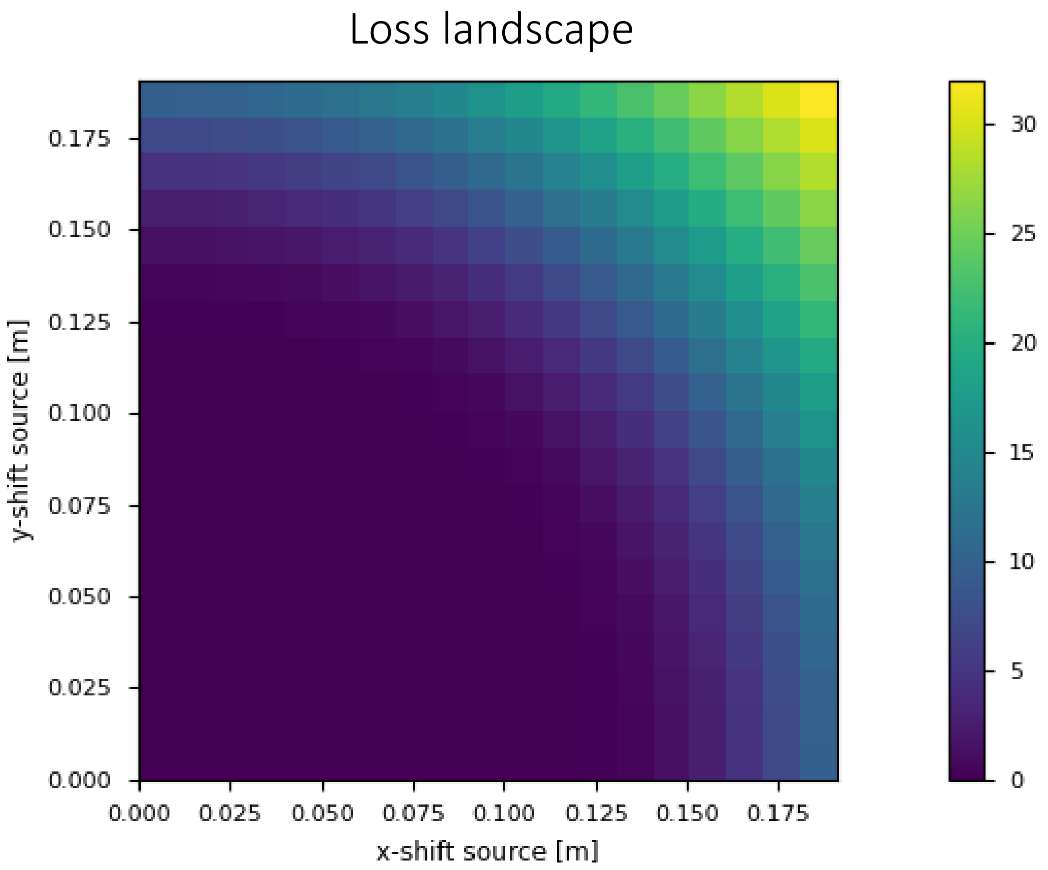

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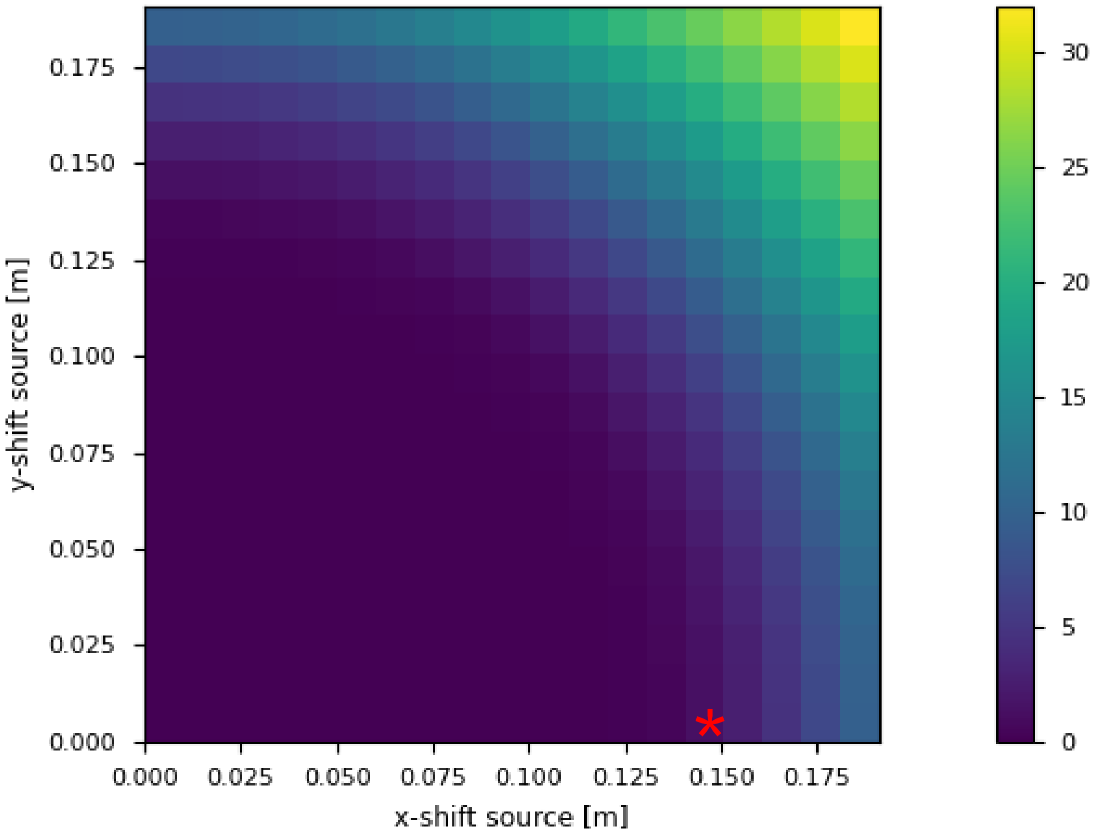

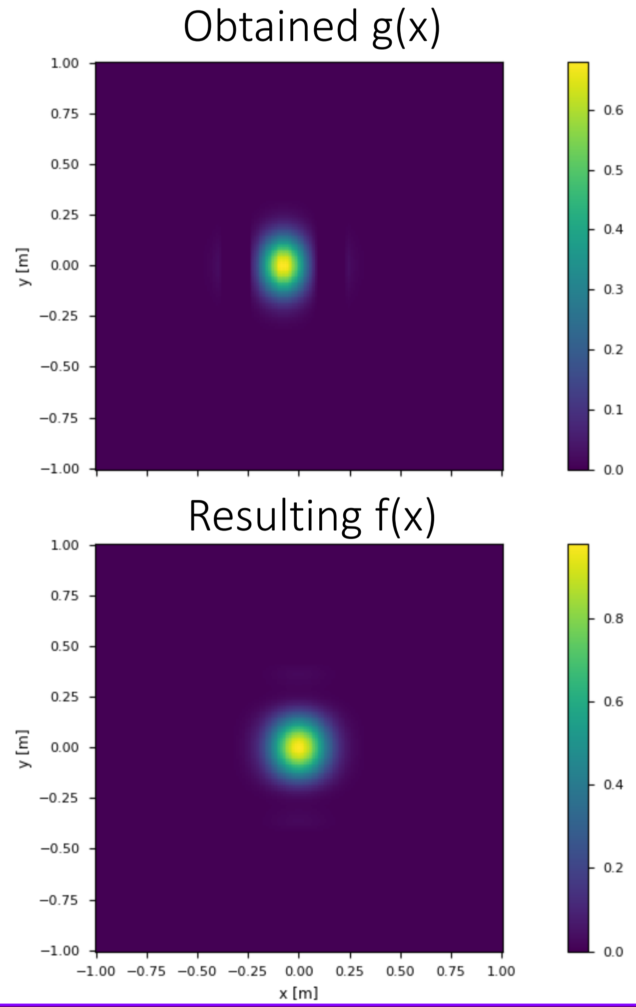

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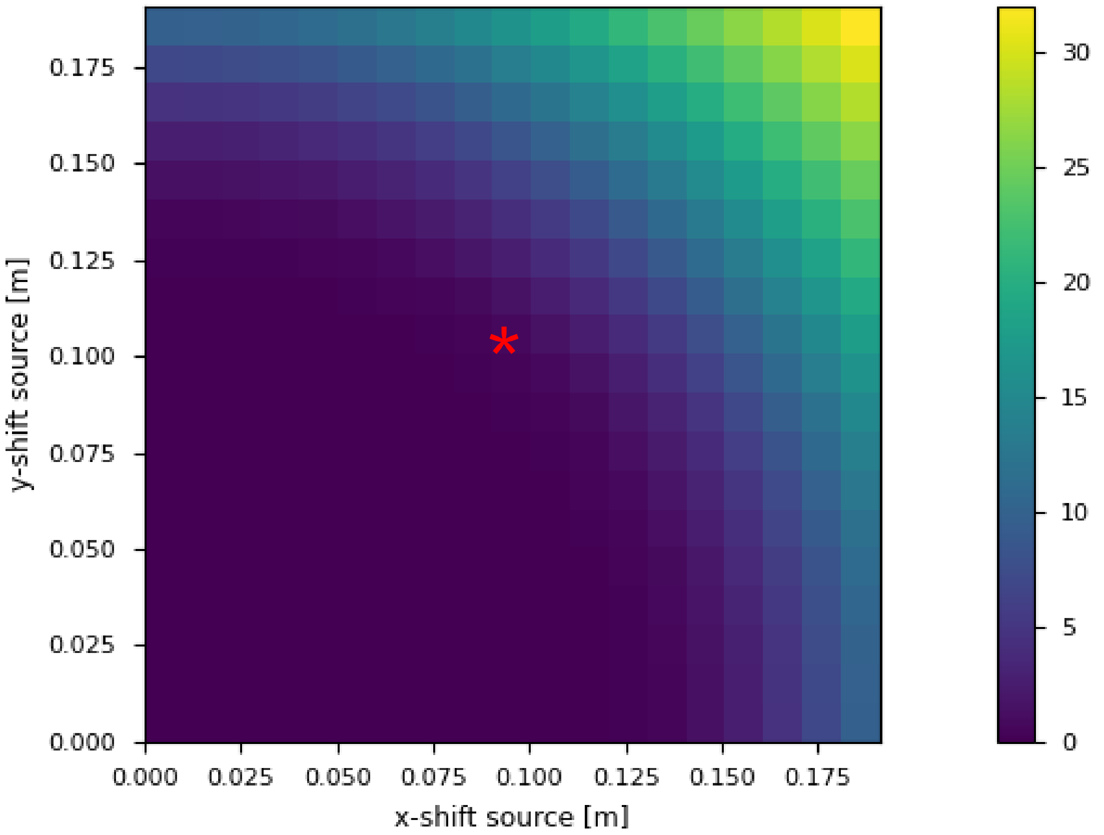

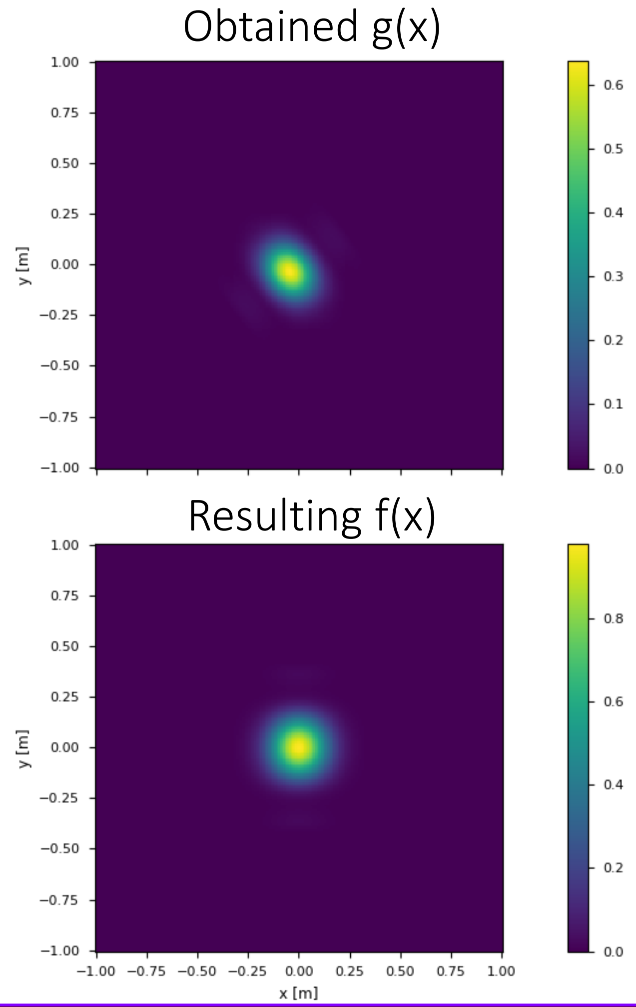

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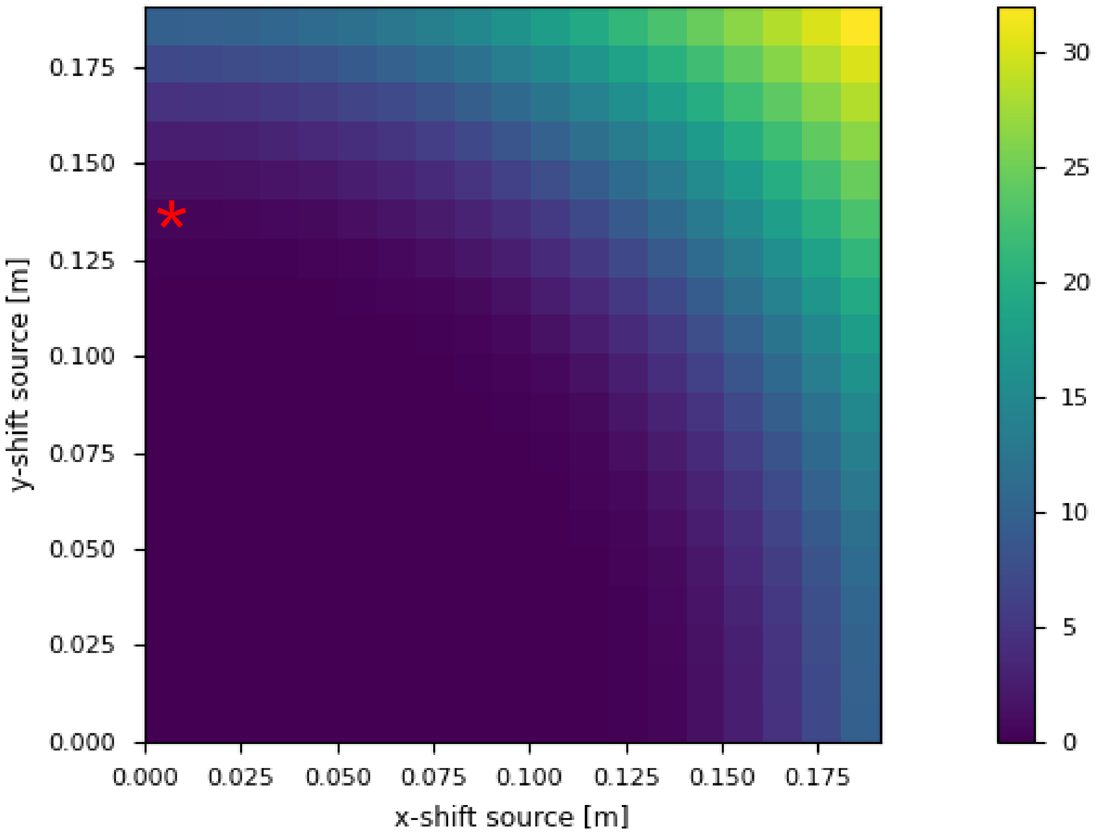
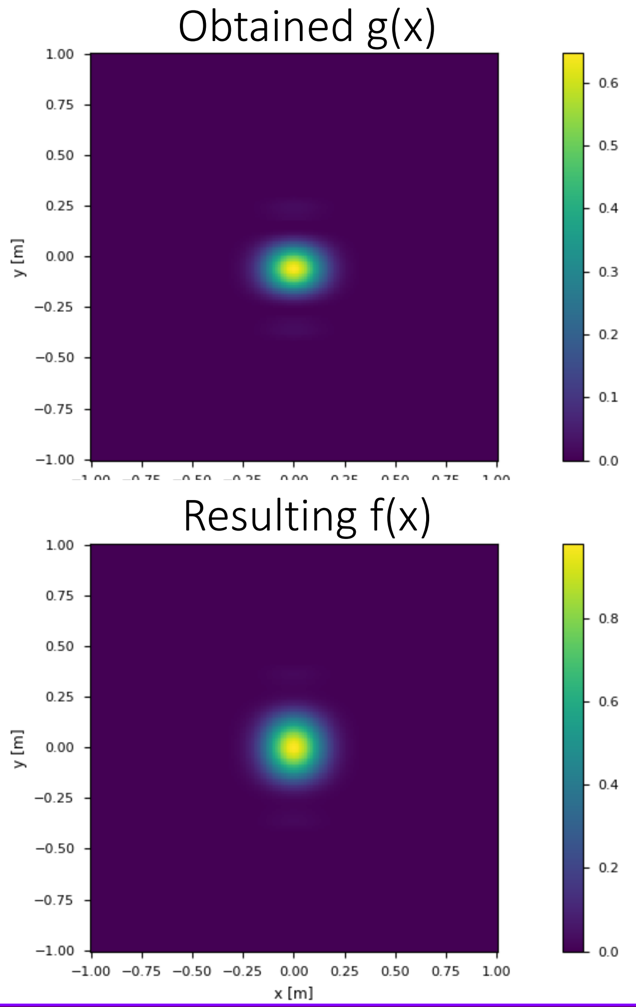
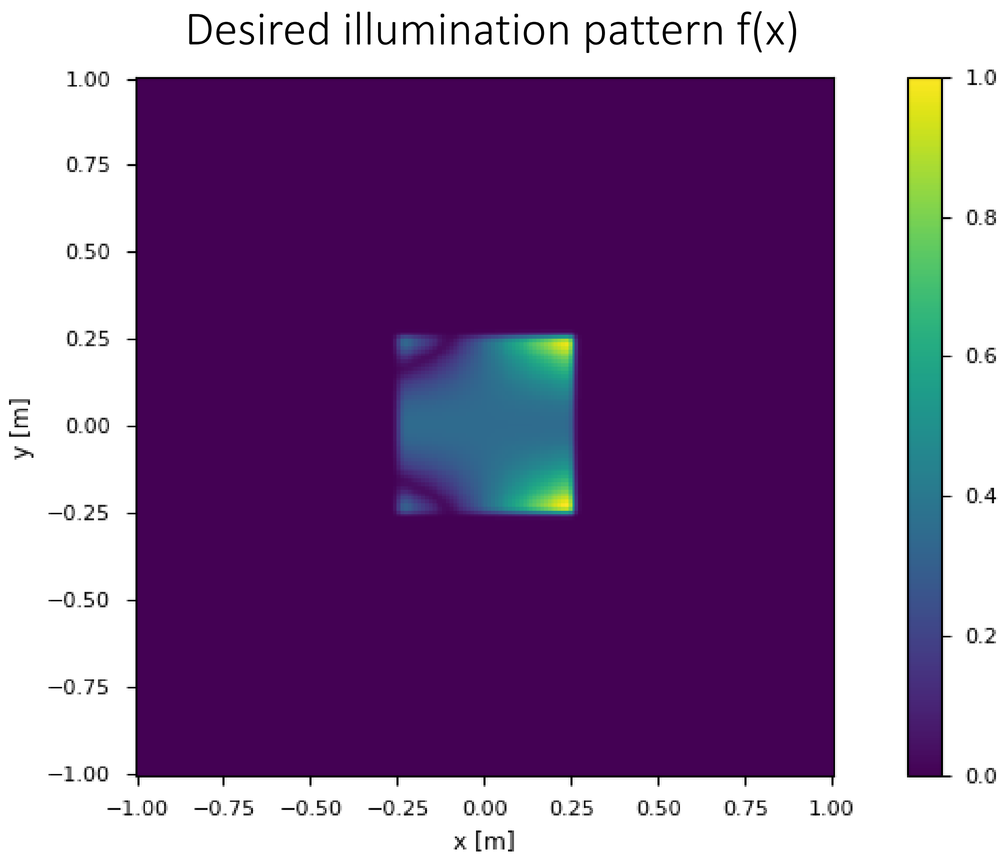
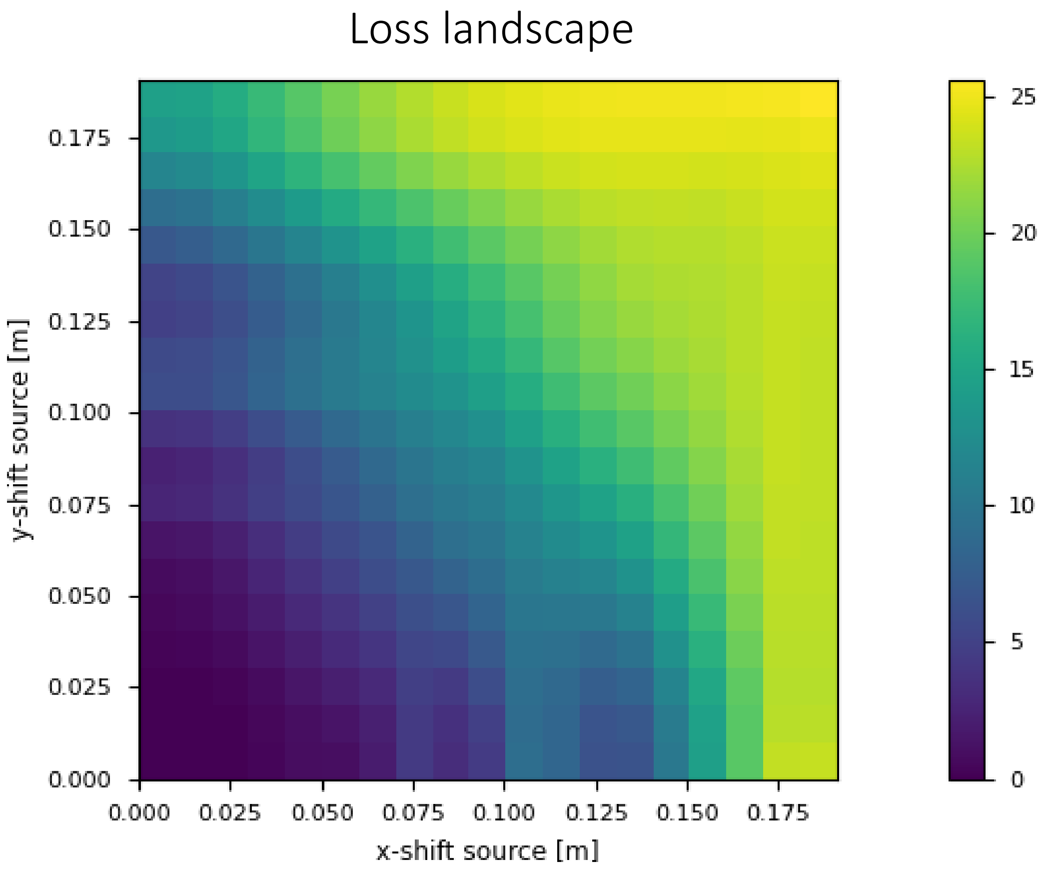




Results 2 Sources





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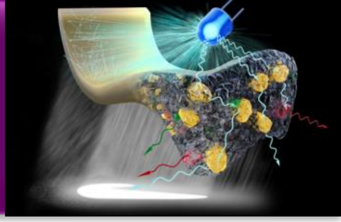

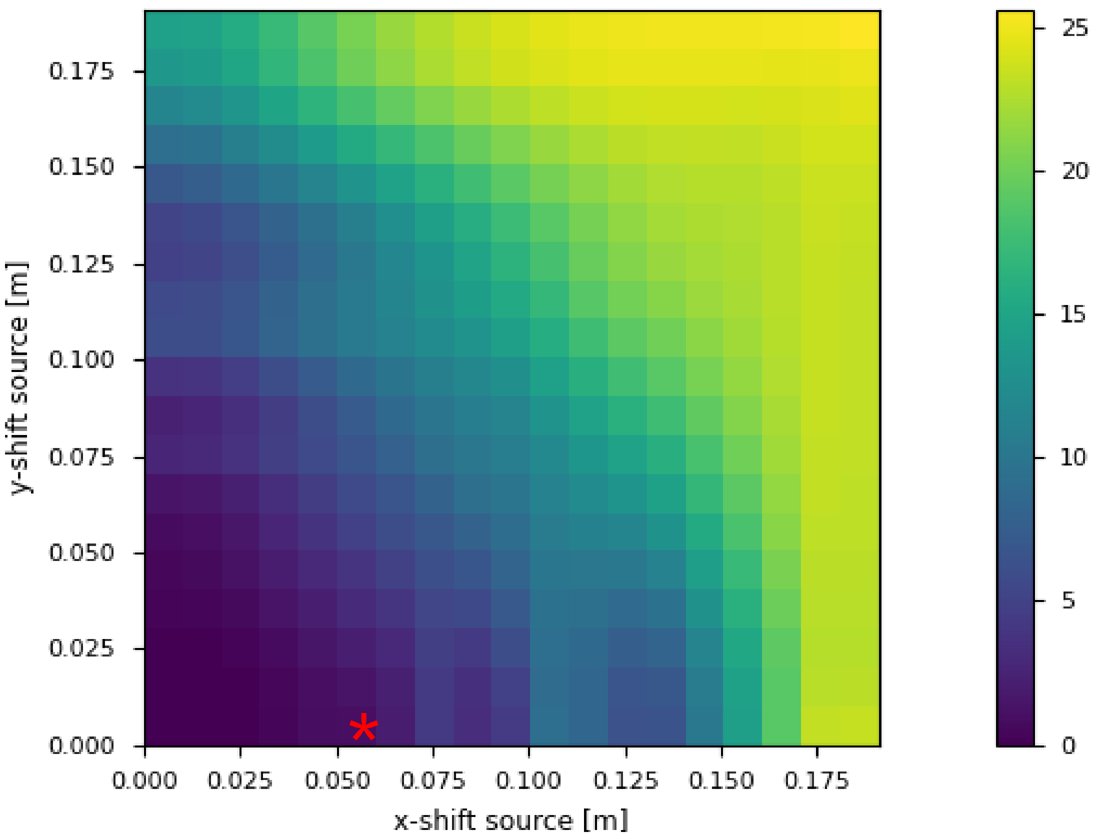

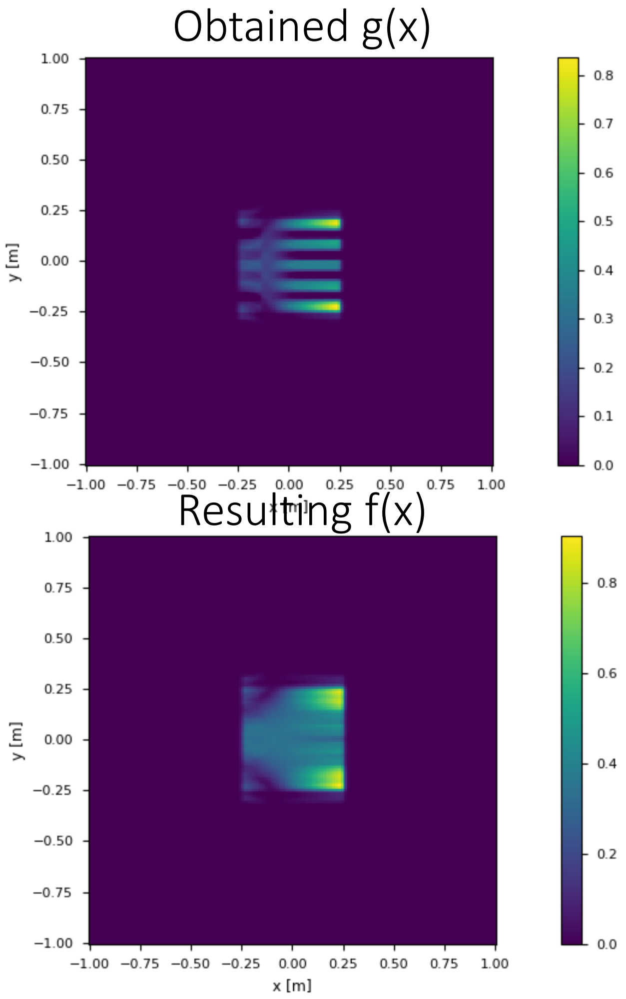

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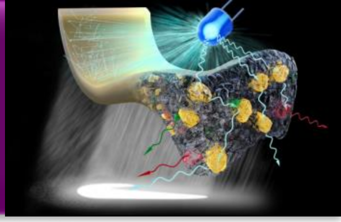
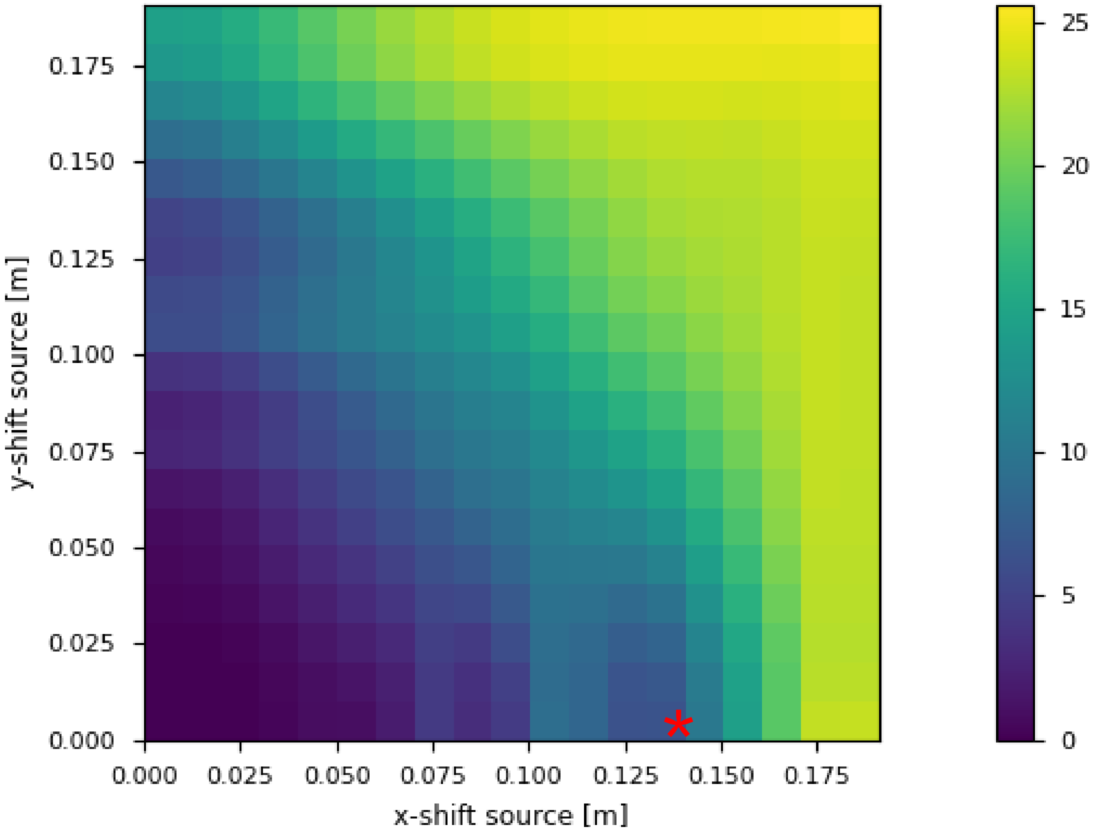
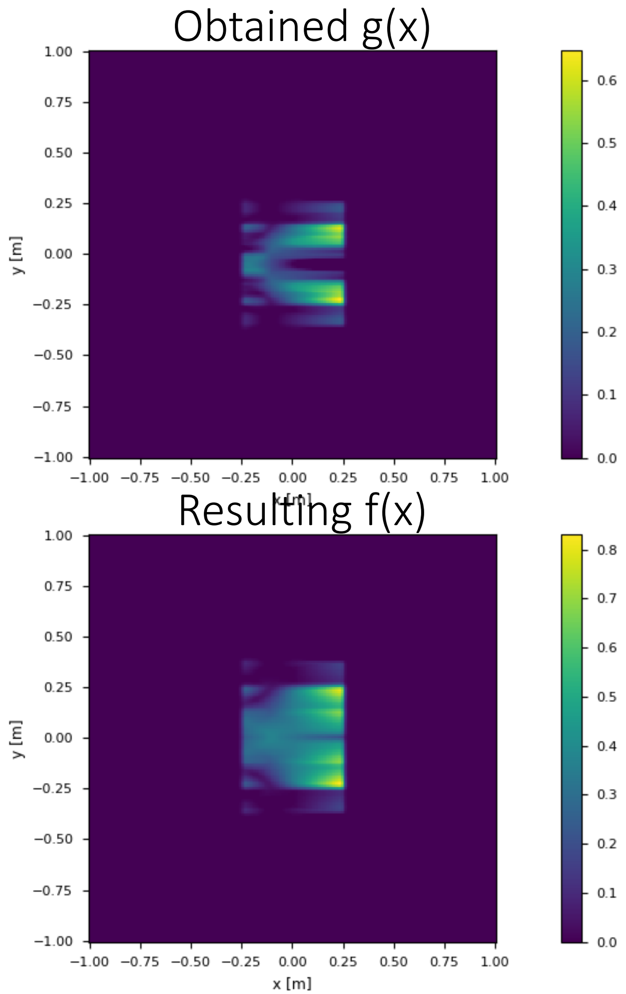
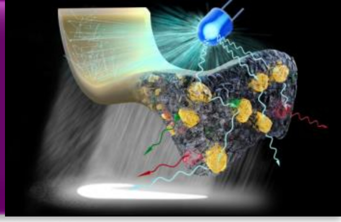






Results 2 Sources



1. Reduce problem to linear problem;
2. Source positions are crucial;
3. Illumination patterns which are “*Refinable*” have a guaranteed solution.